

DIS 0B

1 Writing in Propositional Logic

For each of the following sentences, translate the sentence into propositional logic using the notation introduced in class, and write its negation.

- (a) The square of a nonzero integer is positive.
- (b) There are no integer solutions to the equation $x^2 - y^2 = 10$.
- (c) There is one and only one real solution to the equation $x^3 + x + 1 = 0$.
- (d) For any two distinct real numbers, we can find a rational number in between them.

2 Implication

Which of the following implications are always true, regardless of P ? Give a counterexample for each false assertion.

- (a) $\forall x, \forall y, P(x, y) \implies \forall y, \forall x, P(x, y)$.
- (b) $\exists x, \exists y, P(x, y) \implies \exists y, \exists x, P(x, y)$.
- (c) $\forall x, \exists y, P(x, y) \implies \exists y, \forall x, P(x, y)$.

(d) $\exists x, \forall y, P(x, y) \implies \forall y, \exists x, P(x, y)$.

3 Necessary and Sufficient Conditions

- (a) Given implication $A \implies B$, A is a _____ condition for B .
- (b) Given implication $\neg A \implies \neg B$, A is a _____ condition for B .
- (c) Given implication $\neg B \implies \neg A$, A is a _____ condition for B .
- (d) Given implication $B \implies A$, A is a _____ condition for B .