1 Birthdays

Suppose you record the birthdays of a large group of people, one at a time until you have found a match, i.e., a birthday that has already been recorded. (Assume there are 365 days in a year.)

(a) What is the probability that after the first 3 people’s birthdays are recorded, no match has occurred (i.e. each person has a unique birthday)?

(b) What is the probability that the first 3 people all share the same birthday?

(c) What is the probability that it takes more than 20 people for a match to occur?

(d) What is the probability that it takes exactly 20 people for a match to occur?

(e) Suppose instead that you record the birthdays of a large group of people, one at a time, until you have found a person whose birthday matches your own birthday. What is the probability that it takes exactly 20 people for this to occur?

2 Rain and Wind

The local weather channel just released a statistic for the months of November and December. It said that the probability that it would rain on a windy day is 0.3 and the probability that it would
rain on a non-windy day is 0.8. The probability of a day being windy is 0.2. As a student in EECS 70, you are curious to play around with these numbers. Find the probability that:

(a) A given day is both windy and rainy.

(b) A given day is rainy.

(c) For a given pair of days, exactly one of the two days is rainy.

(d) A given day that is non-rainy is also non-windy.

3 Lie Detector

A lie detector is known to be $\frac{4}{5}$ reliable when the person is guilty and $\frac{9}{10}$ reliable when the person is innocent. If a suspect is chosen from a group of suspects of which only $\frac{1}{100}$ have ever committed a crime, and the test indicates that the person is guilty, what is the probability that he is innocent?