

## DIS 11A

### 1 Binomial Variance

Throw  $n$  balls into  $m$  bins uniformly at random. For a specific ball  $i$ , what is the variance of the number of roommates it has (i.e. the number of other balls that it shares its bin with)?

### 2 Continuous Intro

(a) Is

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

a valid density function? Why or why not? Is it a valid CDF? Why or why not?

(b) Calculate  $\mathbb{E}[X]$  and  $\text{var}(X)$  for  $X$  with the density function

$$f(x) = \begin{cases} 1/\ell, & 0 \leq x \leq \ell, \\ 0, & \text{otherwise.} \end{cases}$$

(c) Suppose  $X$  and  $Y$  are independent and have densities

$$f_X(x) = \begin{cases} 2x, & 0 \leq x \leq 1, \\ 0, & \text{otherwise,} \end{cases}$$
$$f_Y(y) = \begin{cases} 1, & 0 \leq y \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

What is their joint distribution?

(d) Calculate  $\mathbb{E}[XY]$  for the above  $X$  and  $Y$ .

### 3 Continuous Computations

Let  $X$  be a continuous random variable whose pdf is  $cx^3$  (for some constant  $c$ ) in the range  $0 \leq x \leq 1$ , and is 0 outside this range.

(a) Find  $c$ .

(b) Find  $\mathbb{P}[1/3 \leq X \leq 2/3 \mid X \leq 1/2]$ .

(c) Find  $\mathbb{E}(X)$ .

(d) Find  $\text{var}(X)$ .