CS 70 Discrete Mathematics and Probability Theory
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DIS 12A

1 Bayesian Darts

You play a game of darts with your friend. You are better than he is, and the distances of your darts to the center of the target are i.i.d. Uniform$[0,1]$ whereas his are i.i.d. Uniform$[0,2]$. To make the game fair, you agree that you will throw one dart and he will throw two darts. The dart closest to the center wins the game. What is the probability that you will win? *Note*: The distances *from the center of the board* are uniform.

2 Exponential Median

What is the expected value of the median of three i.i.d. exponential variables with parameter λ?

3 Expecting You to Integrate by Parts!

We derived an alternative form for the expected value of a non-negative integer-valued random variable Y, $\mathbb{E}[Y] = \sum_{i=1}^{\infty} \mathbb{P}(Y \geq i)$. In this problem, we will derive the continuous analog of this expression. Throughout this problem, assume X is a *continuous* non-negative random variable with PDF f_X and $\mathbb{E}[X] < \infty$.

(a) Write an expression for $\mathbb{P}(X \geq x)$ in terms of $f_X(x)$ for $x > 0$. This is called the *complementary cumulative distribution function* of X, of the CDF of X. For this problem, we denote this as $\bar{F}_X(x)$. What is $\bar{F}_X(0)$? How about $\bar{F}_X(x)$ as $x \to \infty$?

(b) Use integration by parts on $\mathbb{E}[X] = \int_0^{\infty} x f_X(x)\,\mathrm{d}x$ to derive the expression in question. [*Hint:* What is the antiderivative of f_X?]