CS 70   Discrete Mathematics and Probability Theory

Fall 2017   Kannan Ramchandran and Satish Rao

DIS 14B

1   The Dwinelle Labyrinth

You have decided to take a humanities class this semester, a French class to be specific. Instead of a final exam, your professor has issued a final paper. You must turn in this paper *before* noon to the professor's office on floor 3 in Dwinelle, and it's currently 11:48 a.m.

Let Dwinelle be modeled by the following Markov chain. Instead of rushing to turn it in, we will spend valuable time computing whether or not we *could have* made it. Suppose walking between floors takes 1 minute.
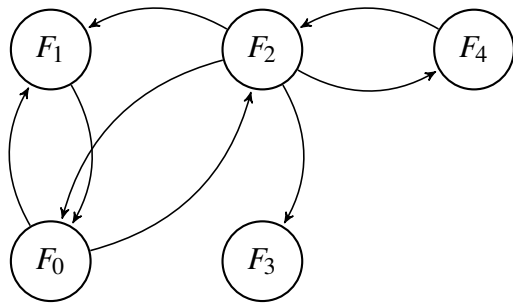
(a) Will you make it in time if you choose a floor to transition to uniformly at random? (If T_i is the number of steps needed to get to F_3 starting from F_i, where $i \in \{0, 1, 2, 3, 4\}$, is $\mathbb{E}[T_0] < 12$?)

(b) Will you make it in time, if for every floor, you order all accessible floors and are twice as likely to take higher floors? (If you are considering 1, 2, or 3, you will take each with probabilities $1/7$, $2/7$, $4/7$, respectively.)

2 Guess the Polynomial

Remember the mantra "$d+1$ points uniquely determine a degree $\leq d$ polynomial ($d \in \mathbb{N}$)"?

Write down a polynomial (of any degree) with non-negative, integer coefficients. Your TA will then guess the exact polynomial you have written down, using only information about the polynomial at two points!

3 Which Envelope?

You have two envelopes in front of you containing cash. You know that one envelope contains twice as much money as the other envelope (the amount of money in an envelope is an integer). You are allowed to pick one envelope and see how much cash is inside, and then based on this information, you can decide to switch envelopes or stick with the envelope you already have.

Can you come up with a strategy which will allow you to pick the envelope with more money, with probability strictly greater than $1/2$?