70: Discrete Math and Probability Theory

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Questions/Announcements

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piazza.com/berkeley/fall2017/cs70

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Programming + Microprocessors

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Programming + Microprocessors $\equiv$ Superpower!

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See note 1, for more discussion.

Wason’s experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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- Suppose you see that Alice went to Baltimore, Bob drove,
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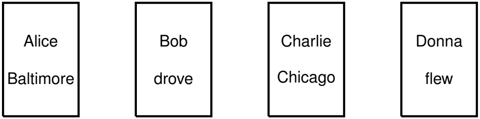

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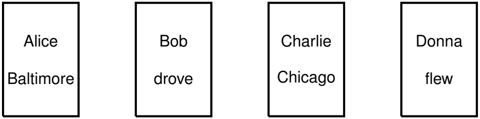

- Which cards must you flip to test the theory?

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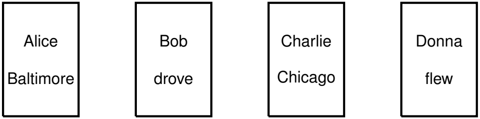

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Answer:

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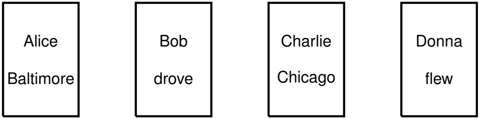

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Answer: Later.

CS70: Lecture 1. Outline.

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The language of proofs!

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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
826th digit of pi is 4
Johny Depp is a good actor
All evens $>$ 2 are sums of 2 primes
$4 + 5$
$x + x$
Alice travelled to Chicago

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$\sqrt{2}$ is irrational	**Proposition**
2+2 = 4	
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Propositional Forms.

Put propositions together to make another...

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Propositional Forms: quick check!

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Put them together..

Propositions:

P_1 - Person 1 rides the bus.

Put them together..

Propositions:

P_1 - Person 1 rides the bus.

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Propositions:
P_1 - Person 1 rides the bus.
P_2 - Person 2 rides the bus.
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But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

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Propositional Form:

$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Put them together..

Propositions:
P_1 - Person 1 rides the bus.
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But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

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Can person 3 ride the bus?

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Can person 3 ride the bus?
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We can program!!!!

We need a way to keep track!

Truth Tables for Propositional Forms.

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
F	F	

Notice: $\wedge$ and $\vee$ are commutative.

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One use for truth tables: Logical Equivalence of propositional forms!

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Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q \qquad\qquad \neg(P \vee Q) \quad \equiv \quad \neg P \wedge \neg Q$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

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Simplify: $(T \wedge Q) \equiv Q$,

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

Distributive?

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

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LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R)$

Distributive?

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

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RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

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P is False .

LHS: $F \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
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P is False .
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Distributive?

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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P is True .

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$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Distributive?

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Simplify: $T \vee Q \equiv T$,

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Foil 1:

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LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
P is False .
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Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$.

Foil 1:
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Foil 2:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

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$(A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)$?

Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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Examples:

Statement: If you stand in the rain, then you'll get wet.

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Conclude: Q is true.

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Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"

Implication.

$P \implies Q$ interpreted as

If P, then Q.

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Conclude: Q is true.

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Statement: If you stand in the rain, then you'll get wet.
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$P \implies Q$ interpreted as

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Statement:
If right triangle has sidelengths $a, b \leq c$, then $a^2 + b^2 = c^2$.

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P = "a right triangle has sidelengths $a \leq b \leq c$",
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Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

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only is False if P is True and Q is False .

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Some Fun: use propositional formulas to describe implication?

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The chemical plant pollutes river. Can we conclude fish die?

Some Fun: use propositional formulas to describe implication?
$((P \implies Q) \wedge P) \implies Q$.

Implication and English.

$P \implies Q$

- If P, then Q.

Implication and English.

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- If P, then Q.
- Q if P.
  Just reversing the order.

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Implication and English.

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- If P, then Q.
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- P only if Q.
  Remember if P is true then Q must be true.
  this suggests that P can only be true if Q is true.
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  this suggests that P can only be true if Q is true.
  since if Q is false P must have been false.
- P is sufficient for Q.
  This means that proving P allows you
  to conclude that Q is true.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

Truth Table: implication.

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T	T	T
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Truth Table: implication.

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Truth Table: implication.

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T	F	F
F	T	T
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P	Q	$\neg P \vee Q$
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T	F	F
F	T	T
F	F	T

$\neg P \vee Q \quad \equiv P \implies Q.$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

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  $P \implies Q \equiv \neg P \vee Q \equiv \neg(\neg Q) \vee \neg P \equiv \neg Q \implies \neg P$.
- Converse of $P \implies Q$ is $Q \implies P$.

  If fish die the plant pollutes.

  Not logically equivalent!
- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
  (Logically Equivalent: $\iff$.)

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Next: Statements about boolean valued functions!!

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Wait! What is $\mathbb{N}$?

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- See note 0 for more!

Lecture ended here. Will continue from here on Friday.