Today.

Finish undecidability.
Today.

Finish undecidability.

Start counting.
Halting Problem.
Halting Problem.

\[ \text{HALT}(P, I) \]
Halting Problem.

\[
\text{\textit{HALT}}(P, I)
\]

\[P \text{- program}\]
Halting Problem.

\[ \text{HALT}(P, I) \]
- \( P \) - program
- \( I \) - input.
Halting Problem.

$HALT(P, I)$

- $P$ - program
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Determines if $P(I)$ ($P$ run on $I$) halts or loops forever.
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Halting Problem.

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Notice:
Need a computer
Halting Problem.

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...with the notion of a stored program!!!!
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Program is a text string.
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Text string can be an input to a program.
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Implement HALT?

HALT(P, I)
P - program
I - input.

Determines if P(I) (P run on I) halts or loops forever.

Run P on I and check!

How long do you wait?

Something about infinity here, maybe?
Implement HALT?

\[ \text{HALT}(P, I) \]
Implement HALT?

\[ \text{HALT}(P, I) \]

\[ P \text{ - program} \]
Implement HALT?

$HALT(P, I)$

- $P$ - program
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Implement HALT?

$$HALT(P, I)$$

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Halt does not exist.
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\[ \text{HALT}(P, I) \]
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$HALT(P, I)$

$P$ - program
Halt does not exist.

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**Theorem:** There is no program HALT.
Halt and Turing.

Proof:

Assume there is a program \( \text{HALT}(\cdot, \cdot) \).

1. If \( \text{Turing}(P) \) goes into an infinite loop, then \( \text{HALT}(P, P) = \text{halts} \).
2. Otherwise, halt immediately.

Assumption: there is a program \( \text{HALT} \).

There is text that "is" the program \( \text{Turing} \).

See above! Can run Turing on Turing!

Does \( \text{Turing}(\text{Turing}) \) halt?

Case 1: \( \text{Turing}(\text{Turing}) \) halts

\[ \Rightarrow \text{HALTS}(\text{Turing}, \text{Turing}) = \text{halts} \]

\[ \Rightarrow \text{Turing}(\text{Turing}) \text{ loops forever.} \]

Case 2: \( \text{Turing}(\text{Turing}) \) loops forever

\[ \Rightarrow \text{HALTS}(\text{Turing}, \text{Turing}) \neq \text{halts} \]

\[ \Rightarrow \text{Turing}(\text{Turing}) \text{ halts.} \]

Contradiction.

Program \( \text{HALT} \) does not exist!

Questions?
Halt and Turing.

**Proof:** Assume there is a program $HALT(\cdot, \cdot)$. 

...
Halt and Turing.

**Proof:** Assume there is a program $HALT(\cdot,\cdot)$.

$Turing(P)$
Halt and Turing.

**Proof:** Assume there is a program $HALT(\cdot, \cdot)$.

**Turing(P)**

1. If $HALT(P, P) = \text{"halts"}$, then go into an infinite loop.
Halt and Turing.

**Proof:** Assume there is a program $HALT(\cdot,\cdot)$.

Turing(P)
1. If $HALT(P,P) =$ “halts”, then go into an infinite loop.
2. Otherwise, halt immediately.
Halt and Turing.

**Proof:** Assume there is a program $HALT(\cdot,\cdot)$.

$Turing(P)$
1. If $HALT(P,P) = \text{"halts"}$, then go into an infinite loop.
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Assumption: there is a program $HALT$.
Halt and Turing.

**Proof:** Assume there is a program $HALT(\cdot,\cdot)$.

Turing($P$)
1. If $HALT(P,P) = \text{"halts"}$, then go into an infinite loop.
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Can run Turing on Turing!
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Does Turing(Turing) halt?
**Halt and Turing.**

**Proof:** Assume there is a program $HALT(\cdot, \cdot)$.

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Can run $Turing$ on $Turing$!

Does $Turing(Turing)$ halt?

Case 1: $Turing(Turing)$ halts
\[ \implies \text{then } HALTS(Turing, Turing) = \text{halts} \]
Halt and Turing.

Proof: Assume there is a program \( HALT(\cdot,\cdot) \).

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1. If \( HALT(P,P) = \text{"halts"} \), then go into an infinite loop.
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Halt and Turing.

**Proof:** Assume there is a program \( \text{HALT}(\cdot,\cdot) \).

\[
\text{Turing}(P) \\
1. \text{If } \text{HALT}(P,P) = \text{"halts"}, \text{ then go into an infinite loop.} \\
2. \text{Otherwise, halt immediately.}
\]

Assumption: there is a program \( \text{HALT} \).
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Case 2: \( \text{Turing}(\text{Turing}) \) loops forever
Halt and Turing.

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Case 1: $\text{Turing}(\text{Turing})$ halts
$\implies$ then $HALTS(\text{Turing, Turing}) = \text{halts}$
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\[ \implies \text{then } \text{HALTS}(\text{Turing}, \text{Turing}) = \text{halts} \]
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Contradiction.
Program HALT does not exist!
Questions?
Halt and Turing.

Proof: Assume there is a program \(\text{HALT}(\cdot,\cdot)\).

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**Case 1:** $Turing(Turing)$ halts
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Contradiction. Program $HALT$ does not exist!
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Questions?
Another view of proof: diagonalization.

Any program is a fixed length string.
Another view of proof: diagonalization.

Any program is a fixed length string. Fixed length strings are enumerable.
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\[ \begin{array}{c|cccc}
   & P_1 & P_2 & P_3 & \cdots \\
\hline
P_1 & H & H & L & \cdots \\
P_2 & L & L & H & \cdots \\
P_3 & L & H & H & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
\end{array} \]

Halt - diagonal.
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Halt - diagonal. Turing - is not Halt. and is different from every $P_i$ on the diagonal.
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<td>...</td>
</tr>
<tr>
<td>$P_3$</td>
<td>L</td>
<td>H</td>
<td>H</td>
<td>...</td>
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<td>...</td>
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</tr>
</tbody>
</table>

Halt - diagonal.  
Turing - is not Halt.  
and is different from every $P_i$ on the diagonal.  
Turing is not on list. Turing is not a program.  
Turing can be constructed from Halt.
Another view of proof: diagonalization.

Any program is a fixed length string.  
Fixed length strings are enumerable.  
Program halts or not any input, which is a string.

\[
\begin{array}{cccc}
  & P_1 & P_2 & P_3 & \ldots \\
  P_1 & H & H & L & \ldots \\
  P_2 & L & L & H & \ldots \\
  P_3 & L & H & H & \ldots \\
  \vdots & \vdots & \vdots & \vdots & \ddots \\
\end{array}
\]

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<thead>
<tr>
<th></th>
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<th>$P_3$</th>
<th>...</th>
</tr>
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<tr>
<td>$P_1$</td>
<td>H</td>
<td>H</td>
<td>L</td>
<td>...</td>
</tr>
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<td>H</td>
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Halt does not exist!
Wow.
Proof play by play.

Assumed HALT($P, I$) existed.
Proof play by play.

Assumed HALT\((P, I)\) existed.

What is \(P\)?
Proof play by play.

Assumed $\text{HALT}(P, I)$ existed.

What is $P$? Text.
Proof play by play.

Assumed $\text{HALT}(P, I)$ existed.

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Assumed $\text{HALT}(P, I)$ existed.

What is $P$? Text.
What is $I$? Text.

What does it mean to have a program $\text{HALT}(P, I)$.
Proof play by play.

Assumed $\text{HALT}(P, I)$ existed.

What is $P$? Text.
What is $I$? Text.

What does it mean to have a program $\text{HALT}(P, I)$.
You have *Text* that is the program $\text{HALT}(P, I)$. 
Proof play by play.

Assumed $\text{HALT}(P, I)$ existed.

What is $P$? Text.
What is $I$? Text.

What does it mean to have a program $\text{HALT}(P, I)$.
You have $Text$ that is the program $\text{HALT}(P, I)$.
Proof play by play.

Assumed $\text{HALT}(P, I)$ existed.

What is $P$? Text.
What is $I$? Text.

What does it mean to have a program $\text{HALT}(P, I)$.

You have Text that is the program $\text{HALT}(P, I)$.

Have ___ that is the program TURING.
Proof play by play.

Assumed $\text{HALT}(P,I)$ existed.

What is $P$? Text.
What is $I$? Text.

What does it mean to have a program $\text{HALT}(P,I)$.
You have $Text$ that is the program $\text{HALT}(P,I)$.
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Proof play by play.

Assumed $\text{HALT}(P, I)$ existed.

What is $P$? Text.
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What does it mean to have a program $\text{HALT}(P, I)$.

You have Text that is the program $\text{HALT}(P, I)$.

Have Text that is the program $\text{TURING}$.
Here it is!!
Proof play by play.

Assumed \( \text{HALT}(P, I) \) existed.

What is \( P \)? Text.

What is \( I \)? Text.

What does it mean to have a program \( \text{HALT}(P, I) \)?

You have \textit{Text} that is the program \( \text{HALT}(P, I) \).

Have \textit{Text} that is the program \textit{TURING}.

Here it is!!

\textit{Turing}(P)
Proof play by play.

Assumed $\text{HALT}(P, I)$ existed.

What is $P$? Text.
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What does it mean to have a program $\text{HALT}(P, I)$.

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Have Text that is the program TURING.
Here it is!!

$\text{Turing}(P)$

1. If $\text{HALT}(P, P) =$“halts”, then go into an infinite loop.
Proof play by play.

Assumed $\text{HALT}(P, I)$ existed.

What is $P$? Text.
What is $I$? Text.

What does it mean to have a program $\text{HALT}(P, I)$.
You have Text that is the program $\text{HALT}(P, I)$.

Have Text that is the program TURING.
Here it is!!

Turing($P$)
1. If $\text{HALT}(P, P) = \text{"halts"}$, then go into an infinite loop.
2. Otherwise, halt immediately.
Proof play by play.

Assumed $\text{HALT}(P,I)$ existed.

What is $P$? Text.
What is $I$? Text.

What does it mean to have a program $\text{HALT}(P,I)$.
You have $Text$ that is the program $\text{HALT}(P,I)$.

Have $Text$ that is the program $\text{TURING}$.
Here it is!!

$\text{Turing}(P)$

1. If $\text{HALT}(P,P) =$“halts”, then go into an infinite loop.
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$\text{Turing}$ “diagonalizes” on list of program.
Proof play by play.

Assumed HALT($P, I$) existed.

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2. Otherwise, halt immediately.

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It is not a program!!!!

$\Rightarrow$ $\text{HALT}$ is not a program.
Proof play by play.

Assumed \( \text{HALT}(P, I) \) existed.

What is \( P \)? Text.
What is \( I \)? Text.

What does it mean to have a program \( \text{HALT}(P, I) \).

You have Text that is the program \( \text{HALT}(P, I) \).

Have Text that is the program TURING.
Here it is!!

\[ \text{Turing}(P) \]
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2. Otherwise, halt immediately.

Turing “diagonalizes” on list of program.
It is not a program!!!!
\[ \implies \text{HALT} \text{ is not a program.} \]

Questions?
We are so smart!

Wow, that was easy!
We are so smart!

Wow, that was easy!
We should be famous!
No computers for Turing!

In Turing’s time.
No computers for Turing!

In Turing’s time.
No computers.
No computers for Turing!

In Turing’s time.
No computers.
Adding machines.
No computers for Turing!

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e.g., Babbage (from table of logarithms) 1812.
No computers for Turing!

In Turing’s time.
No computers.
Adding machines.
e.g., Babbage (from table of logarithms) 1812.
Concept of program as data wasn’t really there.
Turing machine.

A Turing machine – an (infinite) tape with characters – be in a state, and read a character – move left, right, and/or write a character.

Universal Turing machine – an interpreter program for a Turing machine – where the tape could be a description of a ...

Now that's a computer!

Turing: AI, self modifying code, learning...
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Turing: AI, self modifying code, learning...
Turing and computing.

Just a mathematician?
Turing and computing.

Just a mathematician?

“Wrote” a chess program.
Turing and computing.

Just a mathematician?

“Wrote” a chess program.

Simulated the program by hand to play chess.
Turing and computing.

Just a mathematician?
“Wrote” a chess program.
Simulated the program by hand to play chess.
It won!
Just a mathematician?

“Wrote” a chess program.

Simulated the program by hand to play chess.

It won! Once anyway.
Turing and computing.

Just a mathematician?

“Wrote” a chess program.

Simulated the program by hand to play chess.

It won! Once anyway.

Involved with computing labs through the 40s.
Church, Gödel and Turing.

Church proved an equivalent theorem. (Previously.)
Church, Gödel and Turing.

Church proved an equivalent theorem. (Previously.)
Used $\lambda$ calculus....

Gödel: Incompleteness theorem.
Any formal system either is inconsistent or incomplete.
Inconsistent: A false sentence can be proven.
Incomplete: There is no proof for some sentence in the system.
Along the way: "built" computers out of arithmetic.
Showed that every mathematical statement corresponds to.... a natural number!
Same cardinality as.... Text.

Today: Programs can be written in ascii.
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Church, Gödel and Turing.

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Programming languages!
Church, Gödel and Turing.

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Church, Gödel and Turing.

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Today: Programs can be written in ascii.
Computing on top of computing...

Computer, assembly code, programming language, browser, html, javascript..
Computing on top of computing...

Computer, assembly code, programming language, browser, html, javascript..

We can’t get enough of building more Turing machines.
Undecidable problems.

Does a program, \( P \), print “Hello World”? 
Undecidable problems.

Does a program, $P$, print “Hello World”? How?
Undecidable problems.

Does a program, $P$, print “Hello World”? How? What is $P$?
Undecidable problems.

Does a program, $P$, print “Hello World”? How? What is $P$? Text!!!!!
Undecidable problems.

Does a program, $P$, print “Hello World”? How? What is $P$? Text!!!!!!
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Does a program, $P$, print “Hello World”? How? What is $P$? Text!!!!!

Find exit points and add statement: Print “Hello World.”
Undecidable problems.

Does a program, $P$, print “Hello World”? How? What is $P$? Text!!!!!

Find exit points and add statement: Print “Hello World.”
Undecidable problems.

Does a program, $P$, print “Hello World”? How? What is $P$? Text!!!!!!

Find exit points and add statement: **Print** “Hello World.”

Can a set of notched tiles tile the infinite plane?
Undecidable problems.

Does a program, $P$, print “Hello World”? How? What is $P$? Text!!!!!!

Find exit points and add statement: \textbf{Print} “Hello World.”

Can a set of notched tiles tile the infinite plane? Proof: simulate a computer. Halts if finite.
Undecidable problems.

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Does a set of integer equations have a solution?
Undecidable problems.

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Can a set of notched tiles tile the infinite plane? Proof: simulate a computer. Halts if finite.

Does a set of integer equations have a solution? Example: “$x^n + y^n = 1$?”
Undecidable problems.

Does a program, $P$, print “Hello World”? How? What is $P$? Text!!!!!!

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Can a set of notched tiles tile the infinite plane?
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Does a set of integer equations have a solution?
Example: “$x^n + y^n = 1$?”
Problem is undecidable.
Undecidable problems.

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Does a set of integer equations have a solution? Example: “$x^n + y^n = 1$?” Problem is undecidable.

Be careful!
Undecidable problems.

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Problem is undecidable.

Be careful!

Is there an integer solution to $x^n + y^n = 1$?
Undecidable problems.

Does a program, $P$, print “Hello World”? How? What is $P$? Text!!!!!!

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Can a set of notched tiles tile the infinite plane? Proof: simulate a computer. Halts if finite.

Does a set of integer equations have a solution? Example: “ $x^n + y^n = 1$?” Problem is undecidable.

Be careful!

Is there an integer solution to $x^n + y^n = 1$? (Diophantine equation.)
Undecidable problems.

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The answer is yes or no. This “problem” is not undecidable.

Undecidability for Diophantine set of equations
$\implies$ no program can take any set of integer equations and always correctly output whether it has an integer solution.
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This statement is a lie.
Back to technical..

This statement is a lie. *Neither true nor false!*

---

Every person who doesn't shave themselves is shaved by the barber.

Who shaves the barber?

def Turing(P):
    if Halts(P ,P):
        while(true):
            pass
    else:
        return...

Text of Halt...

Halt Program =⇒ Turing Program.

(P =⇒ Q)

Turing("Turing")?

Neither halts nor loops! =⇒ No Turing Program.

No Turing Program =⇒ No halt program.

(¬Q =⇒ ¬P)

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Turing(“Turing”)? Neither halts nor loops! \(\iff\) No Turing program.

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Program is text, so we can pass it to itself, or refer to self.
Summary: decidability.

Computer Programs are an interesting thing.
Computer Programs are an interesting thing. Like Math.
Summary: decidability.

Computer Programs are an interesting thing.
Like Math.
Formal Systems.
Summary: decidability.

Computer Programs are an interesting thing. Like Math. Formal Systems.
Computer Programs are an interesting thing.  
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Computer Programs cannot completely “understand” computer programs.
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Computer Programs cannot completely “understand” computer programs.

Computation is a lens for other action in the world.
Probability

What’s to come?
What’s to come? Probability.
What’s to come? Probability.

A bag contains:
Probability

What’s to come? Probability.

A bag contains:

- Red
- Blue
- Yellow
- Red
- Blue
- Red
- Red
- Blue
What’s to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?
What’s to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?
Count blue.
What’s to come? Probability.

A bag contains:

\[ \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \]

What is the chance that a ball taken from the bag is blue?
Count blue. Count total.
What’s to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?
What’s to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?


For now:
What’s to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?


For now: Counting!
Outline: basics

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn’t matter.
How many outcomes possible for \( k \) coin tosses?
How many poker hands?
How many handshakes for \( n \) people?
How many diagonals in a convex polygon?
How many 10 digit numbers?
How many 10 digit numbers without repetition?
Using a tree..

How many 3-bit strings?
Using a tree..

How many 3-bit strings?
How many different sequences of three bits from \{0, 1\}?
How many 3-bit strings?
How many different sequences of three bits from \( \{0, 1\} \)?
How would you make one sequence?

8 leaves which is \(2 \times 2 \times 2\).

One leaf for each string.

8 3-bit strings!
How many 3-bit strings?
How many different sequences of three bits from \{0, 1\}? How would you make one sequence? How many different ways to do that making?
Using a tree..

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Using a tree..

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How would you make one sequence?
How many different ways to do that making?

```
  0 1
 / \ /
0  1
```

8 leaves which is \(2 \times 2 \times 2\).
One leaf for each string.
8 3-bit strings!
Using a tree..

How many 3-bit strings?
How many different sequences of three bits from \( \{0, 1\} \)?
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```
000
001
010
011
100
101
110
111
```
Using a tree...

How many 3-bit strings?
  How many different sequences of three bits from \( \{0, 1\} \)?
  How would you make one sequence?
  How many different ways to do that making?

- 000
- 001
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How many 3-bit strings?

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How would you make one sequence?

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Using a tree..

How many 3-bit strings?
How many different sequences of three bits from \{0, 1\}?
How would you make one sequence?
How many different ways to do that making?

8 leaves which is $2 \times 2 \times 2$. One leaf for each string.
How many 3-bit strings? How many different sequences of three bits from \(\{0, 1\}\)? How would you make one sequence? How many different ways to do that making?

8 leaves which is \(2 \times 2 \times 2\). One leaf for each string.
Using a tree..

How many 3-bit strings?
How many different sequences of three bits from \{0, 1\}? How would you make one sequence? How many different ways to do that making?

8 leaves which is $2 \times 2 \times 2$. One leaf for each string. 8 3-bit strings!
First Rule of Counting: Product Rule

Objects made by choosing from \( n_1 \), then \( n_2 \), \ldots, then \( n_k \) the number of objects is \( n_1 \times n_2 \times \cdots \times n_k \).
First Rule of Counting: Product Rule

Objects made by choosing from $n_1$, then $n_2$, $\ldots$, then $n_k$ the number of objects is $n_1 \times n_2 \cdots \times n_k$. 
First Rule of Counting: Product Rule

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First Rule of Counting: Product Rule

Objects made by choosing from $n_1$, then $n_2$, ..., then $n_k$ the number of objects is $n_1 \times n_2 \cdots \times n_k$. 

\[
\begin{array}{c}
\times n_1 \\
\times n_2 \\
\times n_3 \\
\vdots \end{array}
\]
First Rule of Counting: Product Rule

Objects made by choosing from \( n_1 \), then \( n_2 \), \ldots, then \( n_k \) the number of objects is \( n_1 \times n_2 \cdots \times n_k \).

In picture, \( 2 \times 2 \times 3 = 12! \)
First Rule of Counting: Product Rule

Objects made by choosing from $n_1$, then $n_2$, ..., then $n_k$ the number of objects is $n_1 \times n_2 \cdots \times n_k$.

In picture, $2 \times 2 \times 3 = 12!$
Using the first rule..

How many outcomes possible for $k$ coin tosses?
Using the first rule..

How many outcomes possible for $k$ coin tosses?

2 ways for first choice,
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2^k$
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2$
Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\( 2 \times 2 \ldots \)
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\[2 \times 2 \cdots \times 2\]
Using the first rule..

How many outcomes possible for $k$ coin tosses?

2 ways for first choice, 2 ways for second choice, ... 
$$2 \times 2 \cdots \times 2 = 2^k$$
Using the first rule..

How many outcomes possible for $k$ coin tosses?

2 ways for first choice, 2 ways for second choice, ...

$2 \times 2 \times \ldots \times 2 = 2^k$

How many 10 digit numbers?
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?
10 ways for first choice,
Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\[ 2 \times 2 \cdots \times 2 = 2^k \]

How many 10 digit numbers?
10 ways for first choice, 10 ways for second choice, ...

Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\[
2 \times 2 \cdots \times 2 = 2^k
\]

How many 10 digit numbers?
10 ways for first choice, 10 ways for second choice, ...
10
How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?
10 ways for first choice, 10 ways for second choice, ...
$10 \times$
Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...\[2 \times 2 \cdots \times 2 = 2^k\]

How many 10 digit numbers?
10 ways for first choice, 10 ways for second choice, ...\[10 \times 10 \cdots\]
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?
10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10$
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?
10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^k$
Using the first rule..

How many outcomes possible for $k$ coin tosses?
2 ways for first choice, 2 ways for second choice, ... 
$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?
10 ways for first choice, 10 ways for second choice, ... 
$10 \times 10 \cdots \times 10 = 10^k$

How many $n$ digit base $m$ numbers?
Using the first rule..

How many outcomes possible for \( k \) coin tosses?
2 ways for first choice, 2 ways for second choice, ...
\[ 2 \times 2 \times \cdots \times 2 = 2^k \]

How many 10 digit numbers?
10 ways for first choice, 10 ways for second choice, ...
\[ 10 \times 10 \times \cdots \times 10 = 10^k \]

How many \( n \) digit base \( m \) numbers?
\( m \) ways for first,
Using the first rule..

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2 ways for first choice, 2 ways for second choice, ...
\[ 2 \times 2 \cdots \times 2 = 2^k \]

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10 ways for first choice, 10 ways for second choice, ...
$10 \times 10 \cdots \times 10 = 10^k$

How many $n$ digit base $m$ numbers?
$m$ ways for first, $m$ ways for second, ...
$m^n$
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?

$|T|$ ways to choose for $f(s_1)$,
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?

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Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?

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$... |T|^{|S|}$
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?

$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...

....$|T|^{|S|}$

How many polynomials of degree $d$ modulo $p$?
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?

$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...

$\ldots |T|^{|S|}$

How many polynomials of degree $d$ modulo $p$?

$p$ ways to choose for first coefficient,
How many functions $f$ mapping $S$ to $T$?

$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...

$\ldots |T|^{|S|}$

How many polynomials of degree $d$ modulo $p$?

$p$ ways to choose for first coefficient, $p$ ways for second, ...
Functions, polynomials.

How many functions $f$ mapping $S$ to $T$?
$|T|$ ways to choose for $f(s_1)$, $|T|$ ways to choose for $f(s_2)$, ...
....$|T|^{|S|}$

How many polynomials of degree $d$ modulo $p$?
$p$ ways to choose for first coefficient, $p$ ways for second, ...
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$p$ values for first point,
Functions, polynomials.

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$p$ ways to choose for first coefficient, $p$ ways for second, ...
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...$p^{d+1}$

Questions?
Permutations.

1 How many 10 digit numbers without repeating a digit?
   (leading zeros are ok.)
   10 ways for first, 9 ways for second, 8 ways for third, ...
   \[10 \times 9 \times 8 \times \cdots \times 1 = 10!\].

1 How many different samples of size \(k\) from \(n\) numbers without replacement.
   \(n\) ways for first choice, \(n - 1\) ways for second, \(n - 2\) choices for third, ...
   \[n \times (n - 1) \times (n - 2) \cdots (n - k + 1) = \frac{n!}{(n - k)!}\].

1 How many orderings of \(n\) objects are there?
   Permutations of \(n\) objects.
   \(n\) ways for first, \(n - 1\) ways for second, \(n - 2\) ways for third, ...
   \[n \times \cdots \times 2 \times 1 = n!\].

\(^1\) By definition: \(0! = 1\).
Permutations.

How many 10 digit numbers \textbf{without repeating a digit}? 

\textit{By definition: }0! = 1.
Permutations.

How many 10 digit numbers without repeating a digit? (leading zeros are ok.)

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How many different samples of size \( k \) from \( n \) numbers without replacement?

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Permutations of \( n \) objects.

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How many different samples of size $k$ from $n$ numbers **without replacement**.

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How many orderings of $n$ objects are there? Permutations of $n$ objects.

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... $n \times (n-1) \times (n-2) \cdots 1 = n!$.

$^1$By definition: $0! = 1$.  

How many one-to-one functions from $|S|$ to $|S|$. 

$|S|$ choices for $f(s_1)$, $|S| - 1$ choices for $f(s_2)$, ... 

So total number is $|S| \times (|S| - 1) \cdots 1 = |S|!$.

A one-to-one function is a permutation!
One-to-One Functions.

How many one-to-one functions from $|S|$ to $|S|$. 

A one-to-one function is a permutation!
How many one-to-one functions from $|S|$ to $|S|$.

$|S|$ choices for $f(s_1)$,
One-to-One Functions.

How many one-to-one functions from $|S|$ to $|S|$.

$|S|$ choices for $f(s_1)$, $|S| - 1$ choices for $f(s_2)$, ...
One-to-One Functions.

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So total number is $|S| \times |S| - 1 \cdots 1 = |S|!$

A one-to-one function is a permutation!
Counting sets..when order doesn’t matter.

How many poker hands?

2 When each unordered object corresponds equal numbers of ordered objects.
Counting sets..when order doesn’t matter.

How many poker hands?

\[52 \times 51 \times 50 \times 49 \times 48\]

\[\text{Number of orderings for a poker hand: } "5!"\]

\[\text{Generic: ways to choose 5 out of 52 possibilities.}\]

\[2\]

\(^2\text{When each unordered object corresponds equal numbers of ordered objects.}\]
Counting sets...when order doesn’t matter.

How many poker hands?

\[ 52 \times 51 \times 50 \times 49 \times 48 \ ??? \]

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When each unordered object corresponds equal numbers of ordered objects.

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2 When each unordered object corresponds equal numbers of ordered objects.
Counting sets...when order doesn’t matter.

How many poker hands?

\[ 52 \times 51 \times 50 \times 49 \times 48 \ ??? \]

Are A, K, Q, 10, J of spades

\[ \frac{52!}{5! \times 47!} \]

Generic: ways to choose 5 out of 52 possibilities.

\[ ^2 \text{When each unordered object corresponds equal numbers of ordered objects.} \]
Counting sets...when order doesn’t matter.

How many poker hands?
\[52 \times 51 \times 50 \times 49 \times 48 \ ???\]

Are A, K, Q, 10, J of spades
and 10, J, Q, K, A of spades

\(^2\text{When each unordered object corresponds equal numbers of ordered objects.}\)
Counting sets..when order doesn’t matter.

How many poker hands?

\[52 \times 51 \times 50 \times 49 \times 48 \ ???\]

Are A, K, Q, 10, J of spades
and 10, J, Q, K, A of spades the same?

\[\text{Second Rule of Counting: } \text{If order doesn't matter count ordered objects and then divide by number of orderings.}\]

\[\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}\]

Can write as...

\[\frac{52!}{5! \times 47!}\]

Generic: ways to choose 5 out of 52 possibilities.

\[\text{When each unordered object corresponds equal numbers of ordered objects.}\]
Counting sets..when order doesn’t matter.

How many poker hands?
\[52 \times 51 \times 50 \times 49 \times 48\] ???

Are \(A, K, Q, 10, J\) of spades and \(10, J, Q, K, A\) of spades the same?

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.\(^2\)

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\(^2\)When each unordered object corresponds equal numbers of ordered objects.
Counting sets..when order doesn’t matter.

How many poker hands?

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Number of orderings for a poker hand: “5!”

$^2$When each unordered object corresponds equal numbers of ordered objects.
Counting sets..when order doesn’t matter.

How many poker hands?

\[52 \times 51 \times 50 \times 49 \times 48 \]  

Are \( A, K, Q, 10, J \) of spades  
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**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.\(^2\)

Number of orderings for a poker hand: “5!”  
(The “!” means factorial, not Exclamation.)

\(^2\)When each unordered object corresponds equal numbers of ordered objects.
Counting sets..when order doesn’t matter.

How many poker hands?

\[ 52 \times 51 \times 50 \times 49 \times 48 \quad ??? \]

Are \( A, K, Q, 10, J \) of spades and \( 10, J, Q, K, A \) of spades the same?

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Number of orderings for a poker hand: “5!”

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\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}
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Counting sets..when order doesn’t matter.

How many poker hands?
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\frac{52 \times 51 \times 50 \times 49 \times 48}{5!} \]

Can write as...

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\frac{52!}{5! \times 47!} \]

**Generic:** ways to choose 5 out of 52 possibilities.

\(^2\)When each unordered object corresponds equal numbers of ordered objects.
Ordered to unordered.

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.
Ordered to unordered.

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.

- How many red nodes (ordered objects)? 9.
- How many red nodes mapped to one blue node? 3.
- How many blue nodes (unordered objects)? 3.
- How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.
- How many poker deals per hand? Map each deal to ordered deal: $5!$.
- How many poker hands? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \div 5!$.
Ordered to unordered.

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.

How many red nodes (ordered objects)?

How many blue nodes (unordered objects)?

How many poker deals?

How many poker hands?
Ordered to unordered.

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.

How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node? 3.

How many blue nodes (unordered objects)? 9

$3 = 3$.

How many poker deals?

$52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

How many poker deals per hand?

Map each deal to ordered deal: $5!$.

How many poker hands?

$52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \div 5!$.

Questions?
Ordered to unordered.

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.

How many red nodes (ordered objects)? 9.
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Ordered to unordered.

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Second Rule of Counting: If order doesn’t matter count ordered objects and then divide by number of orderings.

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How many blue nodes (unordered objects)?
Ordered to unordered.

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.

How many red nodes (ordered objects)? 9.
How many red nodes mapped to one blue node? 3.
How many blue nodes (unordered objects)? $\frac{9}{3}$
Ordered to unordered.

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.

How many red nodes (ordered objects)? 9.
How many red nodes mapped to one blue node? 3.
How many blue nodes (unordered objects)? \( \frac{9}{3} = 3 \).
Ordered to unordered.

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.

How many red nodes (ordered objects)? 9.
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How many poker deals per hand?
Ordered to unordered.

**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.

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Map each deal to ordered deal:
**Second Rule of Counting:** If order doesn’t matter count ordered objects and then divide by number of orderings.

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Map each deal to ordered deal: 5!
Ordered to unordered.

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How many poker hands? \(\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!}\)
Questions?
order doesn’t matter.
order doesn’t matter.

Choose 2 out of $n$?
..order doesn’t matter.

Choose 2 out of $n$?

$$n \times (n - 1)$$
..order doesn’t matter.

Choose 2 out of $n$?

\[
\frac{n \times (n - 1)}{2}
\]
..order doesn’t matter.

Choose 2 out of $n$?

\[
\frac{n \times (n - 1)}{2} = \frac{n!}{(n - 2)! \times 2}
\]
..order doesn’t matter.

Choose 2 out of \( n \)?

\[
\frac{n \times (n - 1)}{2} = \frac{n!}{(n - 2)! \times 2}
\]

Choose 3 out of \( n \)?
..order doesn’t matter.

Choose 2 out of $n$?

\[
\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}
\]

Choose 3 out of $n$?

\[
\frac{n \times (n-1) \times (n-2)}{(n-1) \times (n-2)} = \frac{n!}{(n-3)! \times 3}
\]

Notation: \((\binom{n}{k})\) and pronounced "$n$ choose $k$."
order doesn’t matter.

Choose 2 out of $n$?

$$\frac{n \times (n - 1)}{2} = \frac{n!}{(n - 2)! \times 2}$$

Choose 3 out of $n$?

$$\frac{n \times (n - 1) \times (n - 2)}{3!}$$
..order doesn’t matter.

Choose 2 out of $n$?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

Choose 3 out of $n$?

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Notation: $\binom{n}{k}$ and pronounced "$n$ choose $k$."

Familiar? Questions?
order doesn’t matter.

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\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}
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Choose 3 out of $n$?

\[
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\]

Choose $k$ out of $n$?

\[
\frac{n!}{(n-k)!}
\]
..order doesn’t matter.

Choose 2 out of $n$?

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\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}
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$$\frac{n!}{(n-k)! \times k!}$$

Notation: \( \binom{n}{k} \) and pronounced "$n$ choose $k$."

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$$\frac{n \times (n - 1)}{2} = \frac{n!}{(n - 2)! \times 2}$$

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Choose $k$ out of $n$?

$$\frac{n!}{(n - k)! \times k!}$$

Notation: $\binom{n}{k}$ and pronounced “$n$ choose $k$."

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Notation: $\binom{n}{k}$ and pronounced “$n$ choose $k$.”

Familiar? Questions?
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdot \cdot \cdot \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...
Example: Visualize the proof.

**First rule:** $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**

**Second rule:** when order doesn’t matter divide...

3 card Poker deals: 52
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \ldots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \)
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 \)
Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. 
Example: Visualize the proof..

**First rule:** \( n_1 \times n_2 \cdots \times n_3 \). **Product Rule.**

Second rule: when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Poker hands: \( \Delta \)?
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Poker hands: \( \Delta ? \)
Hand: \( Q, K, A \).
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

\[
\Delta
\]

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Poker hands: \( \Delta \)?
Hand: \( Q, K, A \).
Deals: \( Q, K, A \) :
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Poker hands: \( \Delta \)?
Hand: \( Q, K, A \).
Deals: \( Q, K, A : Q, A, K : \)
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

\[
\Delta \\
\text{...}
\]

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Poker hands: \( \Delta \)?

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Example: Visualize the proof..

**First rule:** \( n_1 \times n_2 \cdots \times n_3 \). **Product Rule.**

**Second rule:** when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Poker hands: \( \Delta \)?

**Hand:** \( Q, K, A \).


\( \Delta = 3 \times 2 \times 1 \)
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

\[ \cdots \quad \Delta \quad \cdots \]

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Poker hands: \( \Delta ? \)

Hand: \( Q, K, A \).
\( \Delta = 3 \times 2 \times 1 \) First rule again.
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn't matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Poker hands: \( \Delta \)?

Hand: \( Q, K, A \).
\( \Delta = 3 \times 2 \times 1 \) First rule again.
Total:
Example: Visualize the proof.

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: $\Delta$?
Hand: $Q, K, A$.
$\Delta = 3 \times 2 \times 1$ First rule again.
Total: $\frac{52!}{49!3!}$
Example: Visualize the proof.

**First rule:** $n_1 \times n_2 \cdots \times n_3$. **Product Rule.**

**Second rule:** when order doesn’t matter divide...

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.

Poker hands: $\Delta$?

Hand: $Q, K, A$.


$\Delta = 3 \times 2 \times 1$ First rule again.

Total: $\frac{52!}{49!3!}$ Second Rule!
Example: Visualize the proof.

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: $\Delta$?
   Hand: $Q, K, A$.
$\Delta = 3 \times 2 \times 1$ First rule again.
Total: $\frac{52!}{49!3!}$ Second Rule!

Choose $k$ out of $n$. 
Example: Visualize the proof.

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Poker hands: \( \Delta \)?
   Hand: \( Q, K, A \).
\( \Delta = 3 \times 2 \times 1 \) First rule again.
Total: \( \frac{52!}{49!3!} \) Second Rule!

Choose \( k \) out of \( n \).
Ordered set: \( \frac{n!}{(n-k)!} \)
Example: Visualize the proof..

**First rule:** \( n_1 \times n_2 \cdots \times n_3 \). **Product Rule.**

**Second rule:** when order doesn’t matter divide...

\[
\cdots \quad \Delta \quad \cdots
\]

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.

Poker hands: \( \Delta \)?

**Hand:** \( Q, K, A \).


\( \Delta = 3 \times 2 \times 1 \) First rule again.

Total: \( \frac{52!}{49!3!} \) Second Rule!

Choose \( k \) out of \( n \).

**Ordered set:** \( \frac{n!}{(n-k)!} \) Orderings of one hand?
Example: Visualize the proof..

**First rule:** \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.

**Second rule:** when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.

Poker hands: \( \Delta \)?

- **Hand:** Q, K, A.

\( \Delta = 3 \times 2 \times 1 \) First rule again.

Total: \( \frac{52!}{49!3!} \) Second Rule!

Choose \( k \) out of \( n \).

Ordered set: \( \frac{n!}{(n-k)!} \) Orderings of one hand? \( k! \)
Example: Visualize the proof.

**First rule:** \( n_1 \times n_2 \cdots \times n_3 \). **Product Rule.**

**Second rule:** when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.

Poker hands: \( \Delta \)?

- **Hand:** Q, K, A.
- **Deals:** Q, K, A : Q, A, K : K, A, Q : K, A, Q : A, K, Q : A, Q, K. \( \Delta = 3 \times 2 \times 1 \) First rule again.

Total: \( \frac{52!}{49!3!} \) Second Rule!

Choose \( k \) out of \( n \).

- Ordered set: \( \frac{n!}{(n-k)!} \) Orderings of one hand? \( k! \) (By first rule!)
Example: Visualize the proof..

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

3 card Poker deals: $52 \times 51 \times 50 = \frac{52!}{49!}$. First rule.
Poker hands: $\Delta$?
Hand: $Q, K, A$.
$\Delta = 3 \times 2 \times 1$ First rule again.
Total: $\frac{52!}{49!3!}$ Second Rule!

Choose $k$ out of $n$.
Ordered set: $\frac{n!}{(n-k)!}$ Orderings of one hand? $k!$ (By first rule!)
$\implies$ Total: $\frac{n!}{(n-k)!k!}$
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.

Second rule: when order doesn’t matter divide...

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.

Poker hands: \( \Delta \)?

Hand: \( Q, K, A \).


\( \Delta = 3 \times 2 \times 1 \) First rule again.

Total: \( \frac{52!}{49!3!} \) Second Rule!

Choose \( k \) out of \( n \).

Ordered set: \( \frac{n!}{(n-k)!} \)

Orderings of one hand? \( k! \) (By first rule!)

\[ \longrightarrow \text{Total: } \frac{n!}{(n-k)!k!} \text{ Second rule.} \]
Example: Visualize the proof..

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

\[ \cdots \Delta \cdots \]

3 card Poker deals: \( 52 \times 51 \times 50 = \frac{52!}{49!} \). First rule.
Poker hands: \( \Delta ? \)
  Hand: Q, K, A.
\( \Delta = 3 \times 2 \times 1 \) First rule again.
Total: \( \frac{52!}{49!3!} \) Second Rule!

Choose \( k \) out of \( n \).
  Ordered set: \( \frac{n!}{(n-k)!} \) Orderings of one hand? \( k! \) (By first rule!)
  \[ \Longrightarrow \] Total: \( \frac{n!}{(n-k)!k!} \) Second rule.
Example: Anagram

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...
Example: Anagram

First rule: $n_1 \times n_2 \ldots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Example: Anagram

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: $7!$
Example: Anagram

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: 7! First rule.
Example: Anagram

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: 7! First rule.
A’s are the same.
Example: Anagram

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: 7! First rule.
A’s are the same.
What is $\Delta$?
Example: Anagram

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: $7!$ First rule.
A’s are the same.
What is $\Delta$?
ANAGRAM
Example: Anagram

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: 7! First rule.
A’s are the same.
What is \( \Delta \)?
ANAGRAM
\( A_1NA_2GRA_3M \),
Example: Anagram

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: 7! First rule.
A’s are the same.
What is \( \Delta \)?
ANAGRAM
\( A_1 NA_2 GRA_3 M, A_2 NA_1 GRA_3 M, \)...
Example: Anagram

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: 7! First rule.
A’s are the same.
What is \( \Delta \)?
ANAGRAM
\( A_1 N A_2 G R A_3 M , A_2 N A_1 G R A_3 M , \ldots \)
Example: Anagram

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: 7! First rule.
A’s are the same.
What is \( \Delta \)?
ANAGRAM
\( A_1 NA_2 GRA_3 M, A_2 NA_1 GRA_3 M, \ldots \)
\( \Delta = 3 \times 2 \times 1 \)
Example: Anagram

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: $7!$ First rule.
A’s are the same.
What is $\Delta$?

$\text{ANAGRAM}$

$A_1NA_2GRA_3M, A_2NA_1GRA_3M, ...$

$\Delta = 3 \times 2 \times 1 = 3!$
Example: Anagram

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: 7! First rule.
A’s are the same.
What is \( \Delta \)?

ANAGRAM
\( A_1NA_2GRA_3M \), \( A_2NA_1GRA_3M \), ...
\( \Delta = 3 \times 2 \times 1 = 3! \) First rule!
Example: Anagram

First rule: $n_1 \times n_2 \cdots \times n_3$. Product Rule.
Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?
Ordered Set: $7!$ First rule.
A’s are the same.
What is $\Delta$?

ANAGRAM
$A_1NA_2GRA_3M$, $A_2NA_1GRA_3M$, ...
$\Delta = 3 \times 2 \times 1 = 3!$ First rule!
$\implies \frac{7!}{3!}$
Example: Anagram

First rule: \( n_1 \times n_2 \cdots \times n_3 \). Product Rule.

Second rule: when order doesn’t matter divide...

Orderings of ANAGRAM?

Ordered Set: 7! First rule.

A’s are the same.

What is \( \Delta \)?

ANAGRAM

\( A_1NA_2GRA_3M, A_2NA_1GRA_3M, \ldots \)

\( \Delta = 3 \times 2 \times 1 = 3! \)  First rule!

\[ \Rightarrow \frac{7!}{3!} \]  Second rule!
Some Practice.

How many orderings of letters of CAT?

3 ways to choose first letter, 2 ways for second, 1 for last.

\[ \Rightarrow 3 \times 2 \times 1 = 3! \] orderings

How many orderings of the letters in ANAGRAM?

Ordered, except for A!

Total orderings of 7 letters.

7!

Total "extra counts" or orderings of three A's?

3!

Total orderings?

7! / 3!

How many orderings of MISSISSIPPI?

4 S's, 4 I's, 2 P's.

11 letters total.

11! ordered objects.

\[ \frac{11!}{4! \times 4! \times 2!} \] ordered objects per "unordered object"
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.

$$\Rightarrow 3 \times 2 \times 1$$
Some Practice.

How many orderings of letters of CAT?

3 ways to choose first letter, 2 ways for second, 1 for last.

$$\Rightarrow 3 \times 2 \times 1 = 3! \text{ orderings}$$
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.

⇒ $3 \times 2 \times 1 = 3!$ orderings

How many orderings of the letters in ANAGRAM?
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.

\[ \Rightarrow 3 \times 2 \times 1 = 3! \text{ orderings} \]

How many orderings of the letters in ANAGRAM?
Ordered,
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.

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How many orderings of the letters in ANAGRAM?
Ordered, except for A!
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.
\[ \Rightarrow 3 \times 2 \times 1 = 3! \text{ orderings} \]

How many orderings of the letters in ANAGRAM?
Ordered, except for A!
\[ \text{total orderings of 7 letters.} \]
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.
$$3 \times 2 \times 1 = 3! \text{ orderings}$$

How many orderings of the letters in ANAGRAM?
Ordered, except for A!

*total orderings of 7 letters. 7!*
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.

⇒ 3 × 2 × 1 = 3! orderings

How many orderings of the letters in ANAGRAM?

Ordered, except for A!

total orderings of 7 letters. 7!
total “extra counts” or orderings of three A’s?
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.
\[\implies 3 \times 2 \times 1 = 3! \text{ orderings} \]

How many orderings of the letters in ANAGRAM?
Ordered, except for A!

- total orderings of 7 letters. 7!
- total “extra counts” or orderings of three A’s? 3!
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.

\[ \Rightarrow 3 \times 2 \times 1 = 3! \text{ orderings} \]

How many orderings of the letters in ANAGRAM?

Ordered, except for A!

total orderings of 7 letters. 7!
total “extra counts” or orderings of three A’s? 3!

Total orderings?
Some Practice.

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3 ways to choose first letter, 2 ways for second, 1 for last.

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Ordered, except for A!

total orderings of 7 letters. 7!
total “extra counts” or orderings of three A’s? 3!

Total orderings? $\frac{7!}{3!}$
Some Practice.

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How many orderings of the letters in ANAGRAM?
Ordered, except for A!

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- total “extra counts” or orderings of three A’s? 3!

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11 letters total.

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\[ 4! \times 4! \times 2! \text{ ordered objects per “unordered object”} \]

\[ \Rightarrow 11! \div 4! \times 4! \times 2! \]
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.

\[ 3 \times 2 \times 1 = 3! \text{ orderings} \]

How many orderings of the letters in ANAGRAM?
Ordered, except for A!

total orderings of 7 letters. \(7!\)
total “extra counts” or orderings of three A’s? \(3!\)

Total orderings? \(\frac{7!}{3!}\)

How many orderings of MISSISSIPPI?
4 S’s, 4 I’s, 2 P’s.
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.

⇒ 3 \times 2 \times 1 = 3! \text{ orderings}

How many orderings of the letters in ANAGRAM?

Ordered, except for A!

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total “extra counts” or orderings of three A’s? 3!

Total orderings? \(\frac{7!}{3!}\)

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Some Practice.

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How many orderings of the letters in ANAGRAM?
Ordered, except for A!

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Total orderings? \[ \frac{7!}{3!} \]

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Some Practice.

How many orderings of letters of CAT?
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\[3 \times 2 \times 1 = 3! \text{ orderings}\]

How many orderings of the letters in ANAGRAM?
Ordered, except for A!

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total “extra counts” or orderings of three A’s? 3!

Total orderings? \[\frac{7!}{3!}\]

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4 S’s, 4 I’s, 2 P’s.
11 letters total.
11! ordered total.
4! \times 4! \times 2! ordered objects per “unordered object”
Some Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways for second, 1 for last.

\[ 3 \times 2 \times 1 = 3! \text{ orderings} \]

How many orderings of the letters in ANAGRAM?
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total orderings of 7 letters. 7!
total “extra counts” or orderings of three A’s? 3!

Total orderings? \( \frac{7!}{3!} \)

How many orderings of MISSISSIPPI?
4 S’s, 4 I’s, 2 P’s.
11 letters total.

11! ordered objects.

4! \times 4! \times 2! \text{ ordered objects per “unordered object”}

\[ \Rightarrow \frac{11!}{4!4!2!}. \]
More counting on Monday.