Today.

More Counting.

The first rule..

Objects made by choosing from n_1, then $n_2, \ldots$, then n_k the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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Objects made by choosing from n_1, then $n_2, \ldots$, then n_k the number of objects is $n_1 \times n_2 \cdots \times n_k$.

How many outcomes possible for k coin tosses?

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How many outcomes possible for k coin tosses?

2 ways for first choice,

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2 ways for first choice, 2 ways for second choice, ...

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2

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How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2×2

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How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots$

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How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2$

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2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

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How many 10 digit numbers?

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10 ways for first choice,

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10 ways for first choice, 10 ways for second choice, ...

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How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
10

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How many 10 digit numbers?

10 ways for first choice, 10 ways for second choice, ...
$10 \times$

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How many n digit base m numbers?

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m ways for first,

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$10 \times 10 \cdots \times 10 = 10^k$

How many n digit base m numbers?

m ways for first, m ways for second, ...
m^n

Counting sets..when order doesn't matter.

How many poker hands?

[1] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$

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Counting sets..when order doesn't matter.

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$52 \times 51 \times 50 \times 49 \times 48$???

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Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

[1] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$???

Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[1]

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Number of orderings for a poker hand: "5!"

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Counting sets..when order doesn't matter.

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Number of orderings for a poker hand: "5!"
(The "!" means factorial, not Exclamation.)

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Number of orderings for a poker hand: "5!"

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

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How many poker hands?

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Number of orderings for a poker hand: "5!"

Can write as...

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

[1] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

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Are $A, K, Q, 10, J$ of spades
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Number of orderings for a poker hand: "5!"

Can write as...

$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

[1] When each unordered object corresponds equal numbers of ordered objects.

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.

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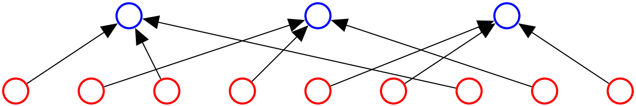

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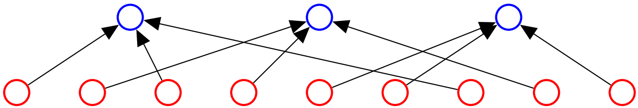

How many red nodes (ordered objects)?

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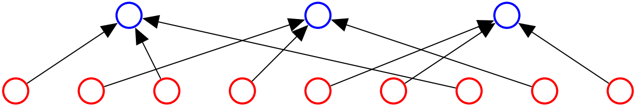

How many red nodes (ordered objects)? 9.

Ordered to unordered.

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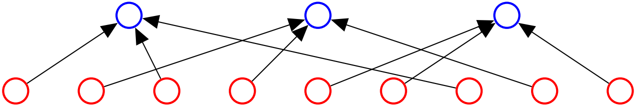

How many red nodes (ordered objects)? 9.

How many red nodes mapped to one blue node?

Ordered to unordered.

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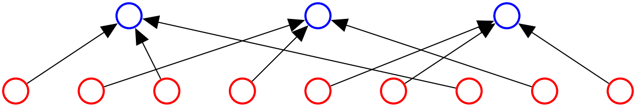

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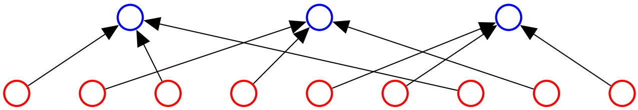

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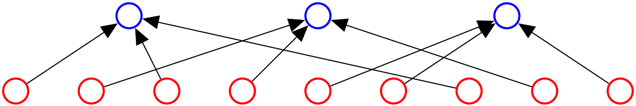

How many red nodes (ordered objects)? 9.

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How many blue nodes (unordered objects)? $\frac{9}{3}$

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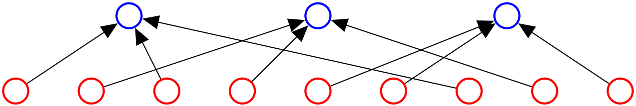

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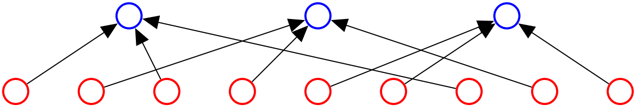

How many red nodes (ordered objects)? 9.

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How many poker deals?

Ordered to unordered.

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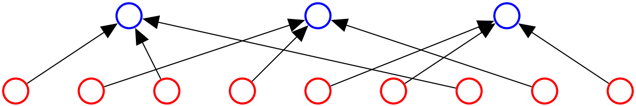

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How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

Ordered to unordered.

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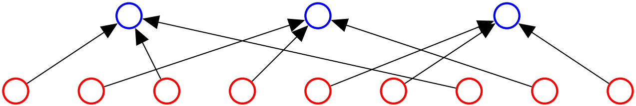

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How many poker deals per hand?

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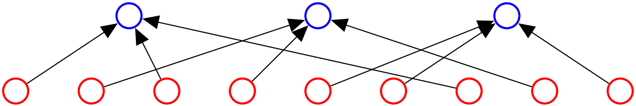
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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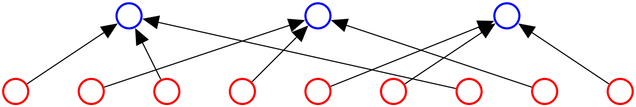
How many poker deals per hand?
  Map each deal to ordered deal:

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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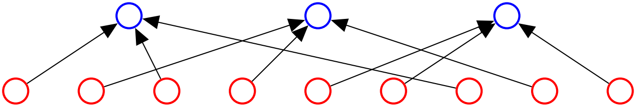
How many blue nodes (unordered objects)? $\frac{9}{3} = 3$.

How many poker deals? $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$.

How many poker deals per hand?
Map each deal to ordered deal: $5!$

Ordered to unordered.

Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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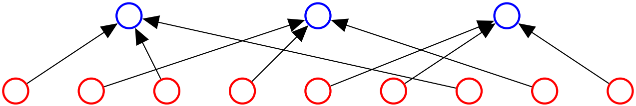

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How many poker deals per hand?
Map each deal to ordered deal: 5!

How many poker hands? $\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!}$

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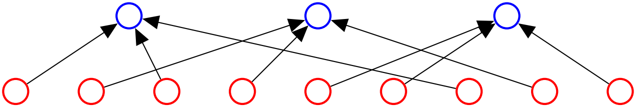
Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.



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Questions?

..order doesn't matter.

..order doesn't matter.

Choose 2 out of n?

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{}$$

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{2}$$

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

Choose 3 out of n?

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

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Choose k **out of** n?

$$\frac{n!}{(n-k)!}$$

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Choose k **out of** n?

$$\frac{n!}{(n-k)! \times k!}$$

Notation: $\binom{n}{k}$ **and pronounced "**n **choose** k**."**

..order doesn't matter.

Choose 2 out of n?

$$\frac{n \times (n-1)}{2} = \frac{n!}{(n-2)! \times 2}$$

Choose 3 out of n?

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Familiar?

..order doesn't matter.

Choose 2 out of n?

$$\frac{n\times(n-1)}{2}=\frac{n!}{(n-2)!\times 2}$$

Choose 3 out of n?

$$\frac{n\times(n-1)\times(n-2)}{3!}=\frac{n!}{(n-3)!\times 3!}$$

Choose k **out of** n?

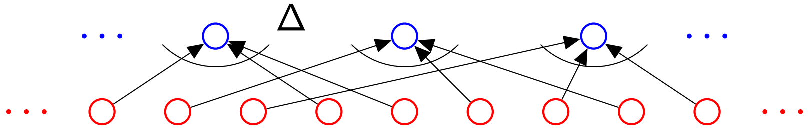
$$\frac{n!}{(n-k)!\times k!}$$

Notation: $\binom{n}{k}$ **and pronounced "n choose k."**

Familiar? Questions?

Example: Anagram

First rule: $n_1 \times n_2 \cdots \times n_3$**. Product Rule.**
Second rule: when order doesn't matter divide...



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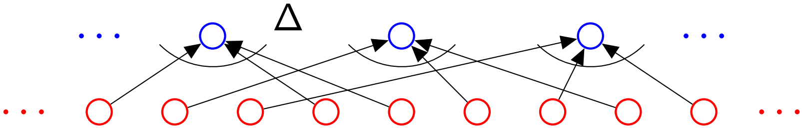

Orderings of ANAGRAM?

Example: Anagram

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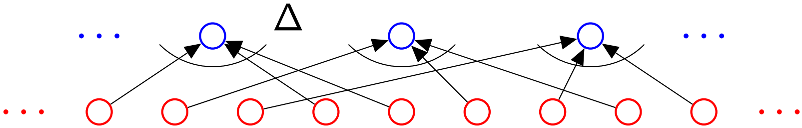

Orderings of ANAGRAM?
Ordered Set: 7!

Example: Anagram

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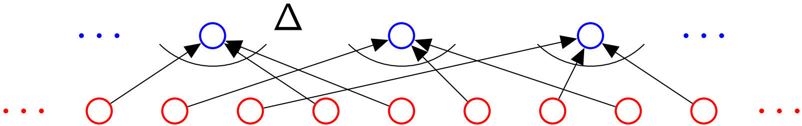

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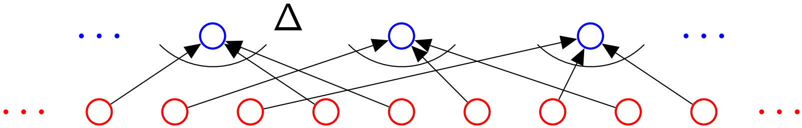

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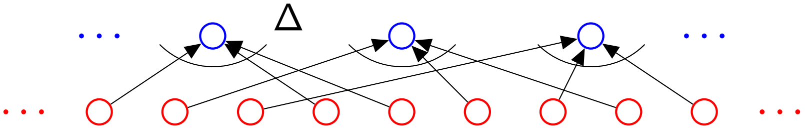

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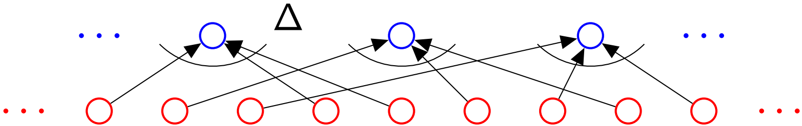

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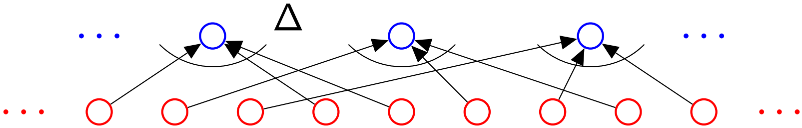

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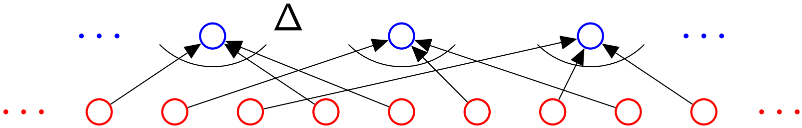

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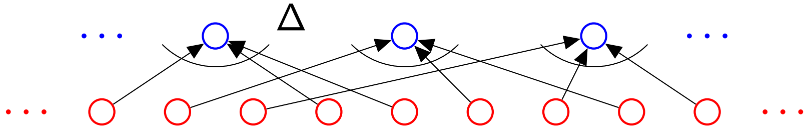

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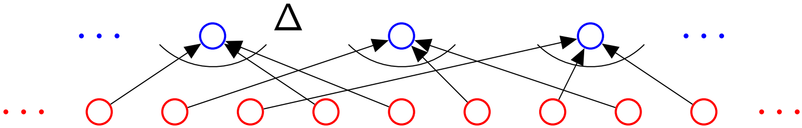

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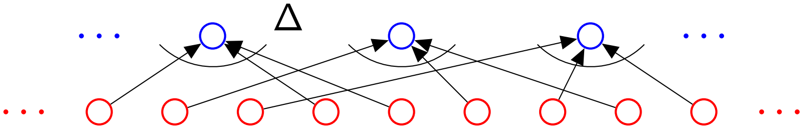

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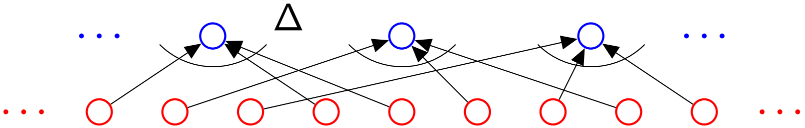

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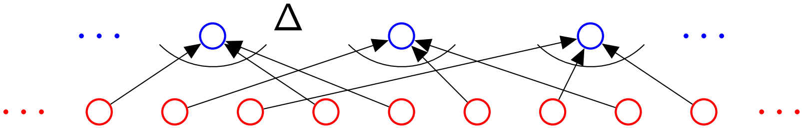

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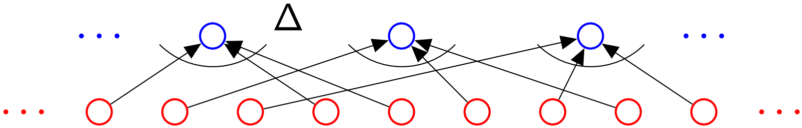

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Some Practice.

How many orderings of letters of CAT?

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3 ways to choose first letter, 2 ways for second, 1 for last.

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How many orderings of MISSISSIPPI?

4 S's, 4 I's, 2 P's.
11 letters total.

Some Practice.

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4 S's, 4 I's, 2 P's.
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11! ordered objects.

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Sample k items out of n

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Without replacement:

Sampling...

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Without replacement:

Order matters:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Sampling...

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- Without replacement:
  - Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$
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    - Second Rule: divide by number of orders

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"n choose k"

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"n choose k"

With Replacement.

Order matters: $n \times n$

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Order matters: $n \times n \times \ldots n = n^k$

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Problem: depends on how many of each item we chose!

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Different number of unordered elts map to each unordered elt.

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Unordered elt: $1,2,3$ $3!$ ordered elts map to it.

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How do we deal with this

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Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "k!"

$\Longrightarrow \frac{n!}{(n-k)!k!}$.

"n choose k"

With Replacement.

Order matters: $n \times n \times \ldots n = n^k$

Order does not matter: Second rule ???

Problem: depends on how many of each item we chose!

Different number of unordered elts map to each unordered elt.

Unordered elt: $1, 2, 3$ — 3! ordered elts map to it.

Unordered elt: $1, 2, 2$ — $\frac{3!}{2!}$ ordered elts map to it.

How do we deal with this mess??

Splitting up some money....

How many ways can Bob and Alice split 5 dollars?

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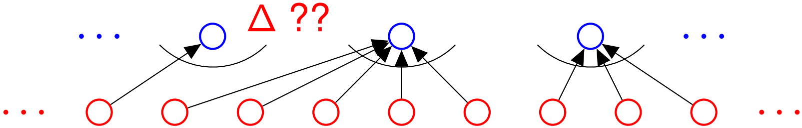

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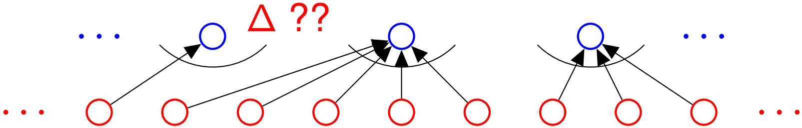

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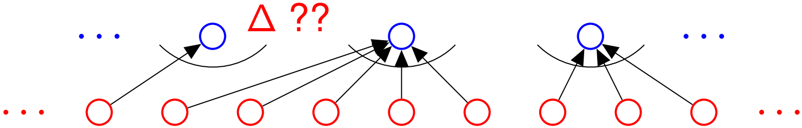

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(A, A, B, B, B): $\binom{5}{2}$;(A,A,B,B,B),(A,B,A,B,B),(A,B,B,A,B),...
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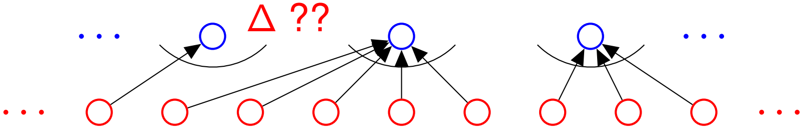

Second rule of counting is no good here!

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Each sequence of stars and bars "is" a split.

Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

Stars and Bars.

How many different 5 star and 2 bar diagrams?

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$| \star | \star \star \star \star$.

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Bars in first and third position.

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Bars in first and third position.

Alice: 1; Bob 4; Eve: 0
$\star | \star \star \star \star |$.
Bars in second and seventh position.

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$\star | \star \star \star \star |$.
Bars in second and seventh position.

$\binom{7}{2}$ ways to do so and
$\binom{7}{2}$ ways to split 5 dollars among 3 people.

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Ways to add up n numbers to sum to k?

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Or: k unordered choices from set of n possibilities with replacement.
Sample with replacement where order doesn't matter.

Quick review of the basics.

First rule: $n_1 \times n_2 \cdots \times n_3$.

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One-to-one rule: equal in number if one-to-one correspondence.

Sample with replacement and order doesn't matter: $\binom{k+n-1}{n-1}$.

Balls in bins.

"k Balls in n bins" $\equiv$ "k samples from n possibilities."

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Example: Poker hands.

5 indistinguishable balls into 3 bins
5 samples from 3 possibilities with replacement and no order

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Example: Poker hands.

5 indistinguishable balls into 3 bins
5 samples from 3 possibilities with replacement and no order
Dividing 5 dollars among Alice, Bob and Eve.

Sum Rule

Two indistinguishable jokers in 54 card deck.
How many 5 card poker hands?

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How many 5 card poker hands? Choose 4 cards plus one of 2 jokers!

$$\binom{52}{5} + 2 * \binom{52}{4} +$$

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Wait a minute!

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Two distinguishable jokers in 54 card deck.
How many 5 card poker hands? Choose 4 cards plus one of 2 jokers!

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Theorem: $\binom{n}{k} = \binom{n}{n-k}$

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How many subsets of size k?
Choose a subset of size $n-k$
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Choosing a subset of size k is same

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$\square$

Pascal’s Triangle

Pascal's Triangle

0

1 1

Pascal's Triangle

		0		
	1		1	
1		2		1

Pascal's Triangle

			0			
		1		1		
	1		2		1	
1		3		3		1

Pascal's Triangle

				0				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

Pascal's Triangle

				0				
			1		1			
		1		2		1		
	1		3		3		1	
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Pascal's Triangle

$$\begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Row n: coefficients of $(1+x)^n = (1+x)(1+x)\cdots(1+x)$.

Pascal's Triangle

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Foil (4 terms)

Pascal's Triangle

0
1 1
1 2 1
1 3 3 1
1 4 6 4 1

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Foil (4 terms) on steroids:

Pascal's Triangle

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 & & & 1 & & 1 & & & \\
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Simplify: collect all terms corresponding to x^k.

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Coefficient of x^k is $\binom{n}{k}$: choose k factors where x is in product.

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 & & & & 0 & & & & \\
 & & & 1 & & 1 & & & \\
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$$
\begin{array}{ccc}
 & \binom{0}{0} & \\
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\end{array}
$$

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Pascal's rule $\implies \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

Combinatorial Proofs.

Theorem: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.
Proof: How many size k subsets of $n+1$?

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Proof: How many size k subsets of $n+1$? $\binom{n+1}{k}$.

How many size k subsets of $n+1$?
How many contain the first element?

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Chose first element,

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Theorem: ${n+1 \choose k} = {n \choose k} + {n \choose k-1}$.

Proof: How many size k subsets of $n+1$? ${n+1 \choose k}$.

How many size k subsets of $n+1$?
How many contain the first element?
Chose first element, need to choose $k-1$ more from remaining n elements.

Combinatorial Proofs.

Theorem: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.
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How many size k subsets of $n+1$?
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How many don't contain the first element ?

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Need to choose k elements from remaining n elts.

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So, $\binom{n}{k-1} + \binom{n}{k}$

Combinatorial Proofs.

Theorem: $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$.

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So, $\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$. $\square$

Combinatorial Proof.

Theorem: $\binom{n}{k} = \binom{n-1}{k-1} + \cdots + \binom{k-1}{k-1}$.

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$$\{1, \ldots, i, \underline{\ldots, n}\}$$

Must choose $k-1$ elements from $n-i$ remaining elements.

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Must choose $k-1$ elements from $n-i$ remaining elements.
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Add them up to get the total number of subsets of size k

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Add them up to get the total number of subsets of size k
which is also $\binom{n+1}{k}$.

$\square$

Binomial Theorem: $x = 1$

Theorem: $2^n = \binom{n}{n} + \binom{n}{n-1} + \cdots + \binom{n}{0}$

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Construct a subset with sequence of n choices:

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Construct a subset with sequence of n choices:
element i **is in** or **is not** in the subset: 2 poss.

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First rule of counting: $2 \times 2 \cdots \times 2 = 2^n$ subsets.

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Theorem: $2^n = \binom{n}{n} + \binom{n}{n-1} + \cdots + \binom{n}{0}$

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Answer: $|S| + |T| - |S \cap T| = 10^9 + 10^9 - 10^8$.

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