CS70: On to probability.

Modeling Uncertainty: Probability Space

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Modeling Uncertainty: Probability Space

1. Key Points
2. Random Experiments
3. Probability Space

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- Probability
  - Models knowledge about uncertainty: Mathematical discipline that allows you to **reason about uncertainty**.
  - Discovers best way to use that knowledge in making decisions

The Magic of Probability

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Uncertainty:

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Uncertainty: vague,

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- Possible outcomes: Heads (H)

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What do we mean by the likelihood of tails is 50%?

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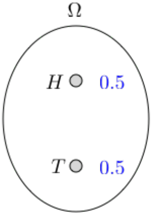
Physical Experiment



Probability Model

Random Experiment: Flip one Fair Coin

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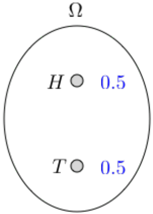

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- The physical experiment is complex.

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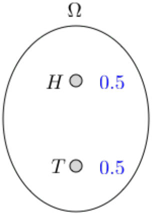

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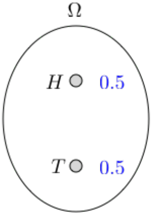

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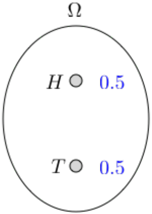

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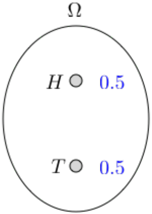

Probability Model

- The physical experiment is complex. (Shape, density, initial momentum and position, ...)
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  - A probability assigned to each outcome: $Pr[H] = 0.5, Pr[T] = 0.5$.

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Flip an unfair (biased, loaded) coin:

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H: 45%

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- Question: How can one figure out p? Flip many times
- Tautology? No: Statistical regularity!

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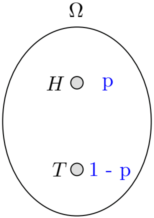

Probability Model

Flip Two Fair Coins

Flip Two Fair Coins

- Possible outcomes:

Flip Two Fair Coins

- Possible outcomes: $\{HH, HT, TH, TT\}$

Flip Two Fair Coins

- Possible outcomes: $\{HH, HT, TH, TT\} \equiv \{H, T\}^2$.

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Flip Glued Coins

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Flips two coins glued together side by side:

Flip Glued Coins

Flips two coins glued together side by side:

Glued coins

50%

50%

Flip Glued Coins

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- Possible outcomes:

Flip Glued Coins

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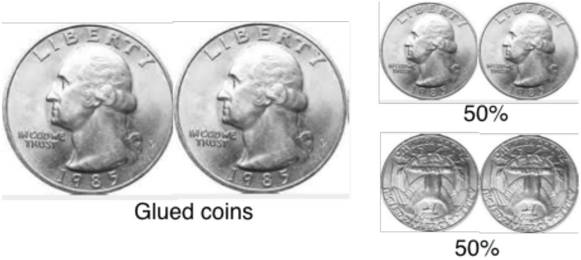

- Possible outcomes: $\{HH, TT\}$.

Flip Glued Coins

Flips two coins glued together side by side:

Glued coins

50%

50%

- Possible outcomes: $\{HH, TT\}$.
- Likelihoods:

Flip Glued Coins

Flips two coins glued together side by side:

Glued coins


50%


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- Possible outcomes: $\{HH, TT\}$.
- Likelihoods: $HH : 0.5, TT : 0.5$.

Flip Glued Coins

Flips two coins glued together side by side:

Glued coins

50%

50%

- Possible outcomes: $\{HH, TT\}$.
- Likelihoods: $HH : 0.5, TT : 0.5$.
- Note: Coins are glued so that they show the same face.

Flip Glued Coins

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Flip Glued Coins

Flips two coins glued together side by side:

- Possible outcomes: $\{HT, TH\}$.
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- Note: Coins are glued so that they show different faces.

Flip two Attached Coins

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Flips two coins attached by a spring:

Flip two Attached Coins

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- Possible outcomes: $\{HH, HT, TH, TT\}$.
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Flip two Attached Coins

Flips two coins attached by a spring:

- Possible outcomes: $\{HH, HT, TH, TT\}$.
- Likelihoods: $HH: 0.4, HT: 0.1, TH: 0.1, TT: 0.4$.

Flip two Attached Coins

Flips two coins attached by a spring:

- Possible outcomes: $\{HH, HT, TH, TT\}$.
- Likelihoods: $HH : 0.4, HT : 0.1, TH : 0.1, TT : 0.4$.
- Note: Coins are attached so that they tend to show the same face, unless the spring twists enough.

Flipping Two Coins

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Here is a way to summarize the four random experiments:

Flipping Two Coins

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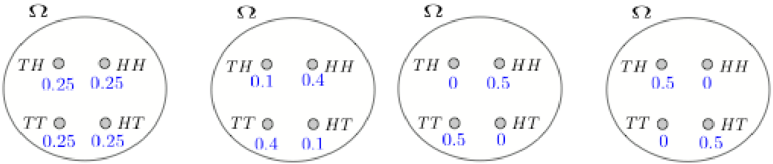

[1] [2] [3] [4]

Flipping Two Coins

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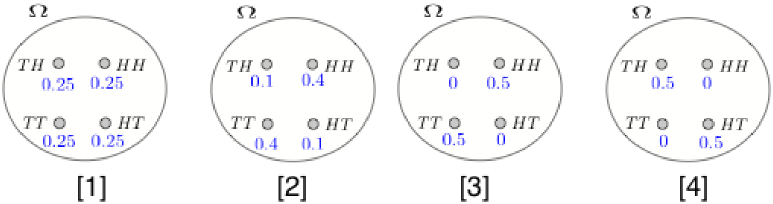

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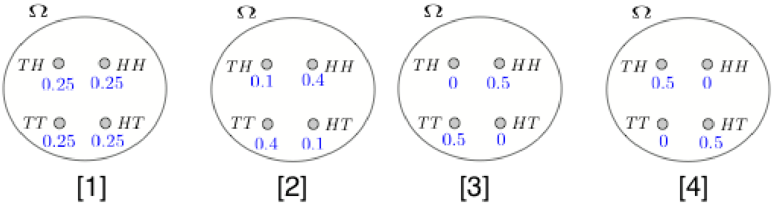

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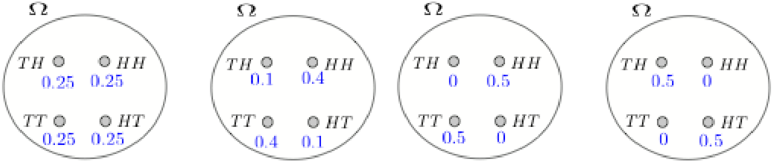

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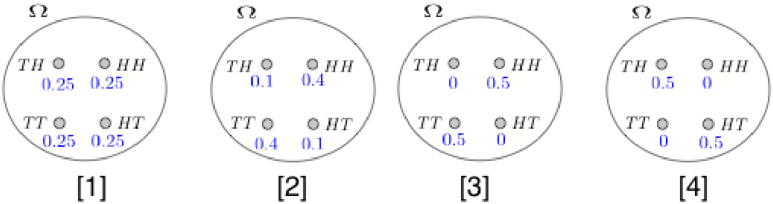

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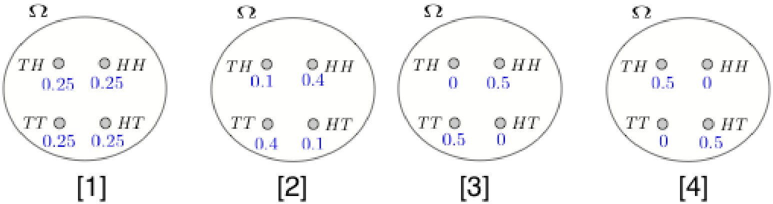

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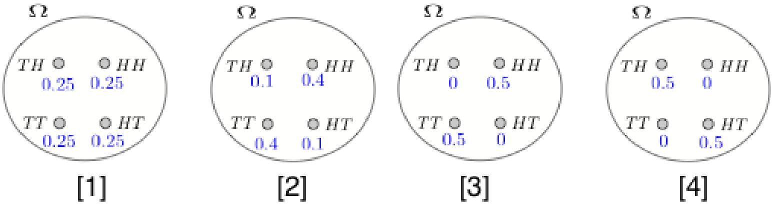

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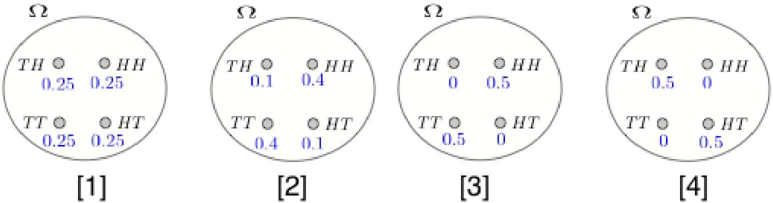

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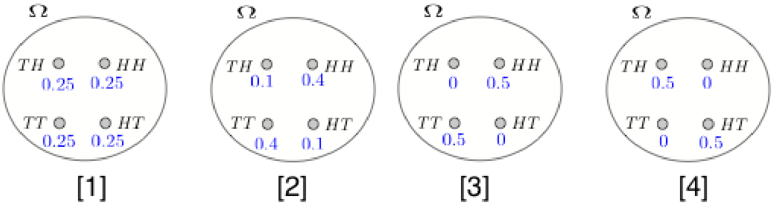

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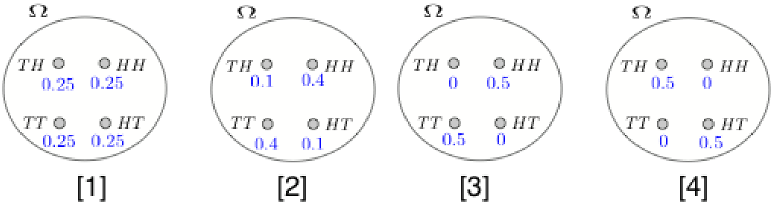

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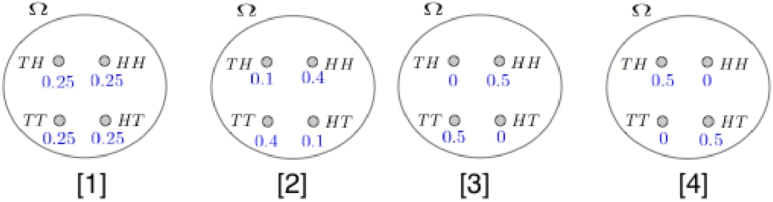

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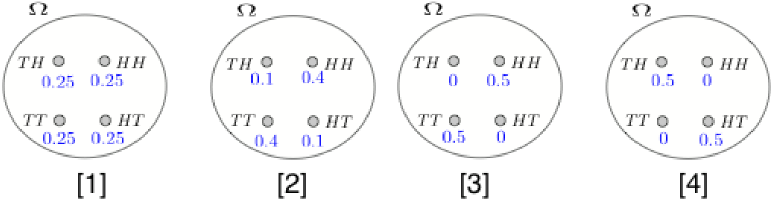

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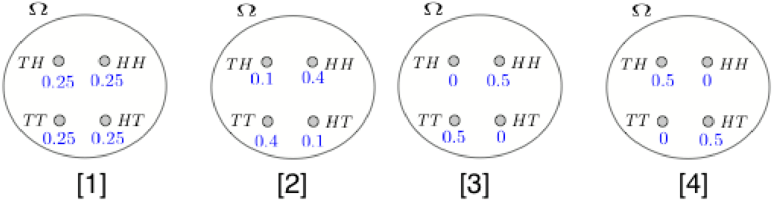

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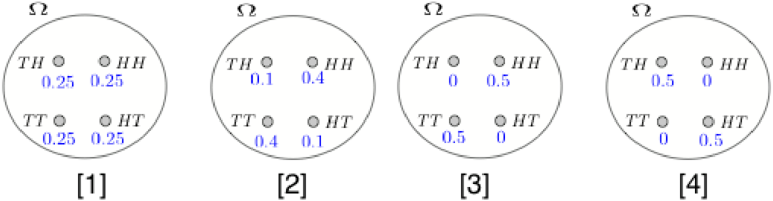

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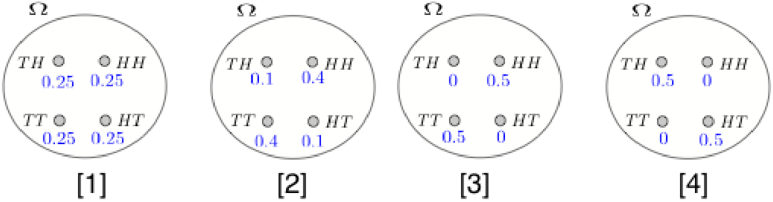

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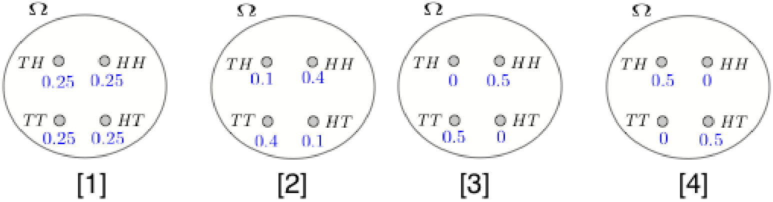

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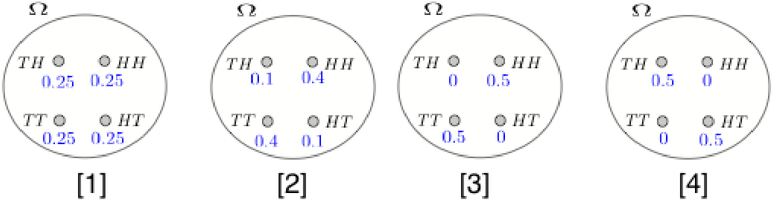

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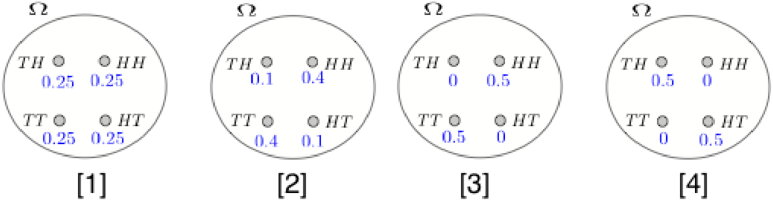

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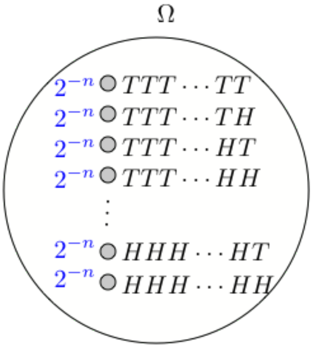

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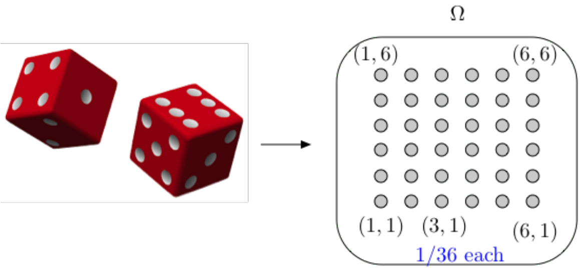

Physical Experiment

Probability Model

Probability Space.

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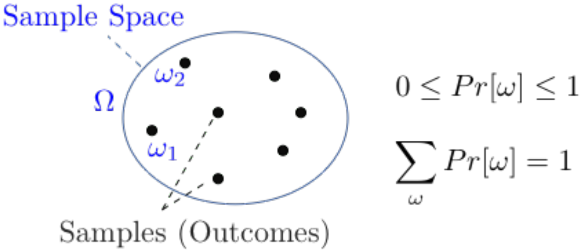

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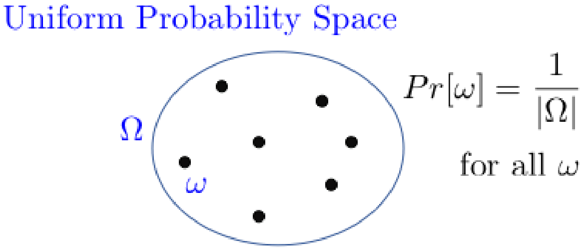

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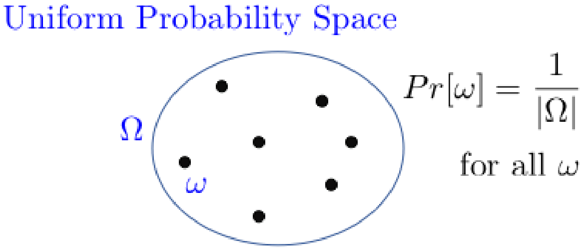

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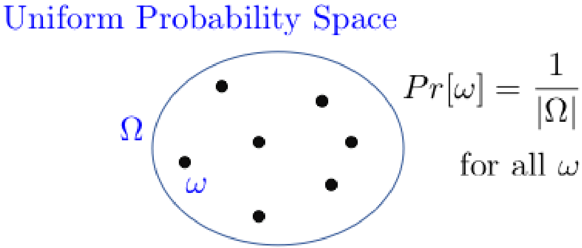

Examples:

- Flipping two fair coins, dealing a poker hand are uniform probability spaces.
- Flipping a biased coin is not a uniform probability space.

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Simplest physical model of a uniform probability space:

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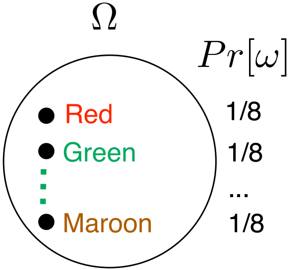
Simplest physical model of a uniform probability space:

Physical experiment



Probability model

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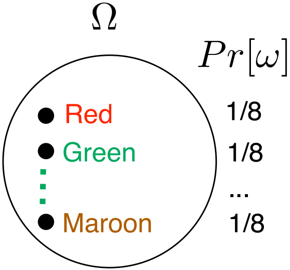

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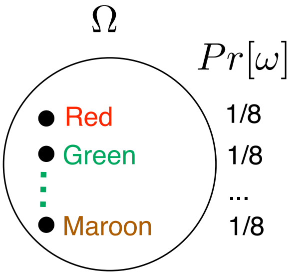

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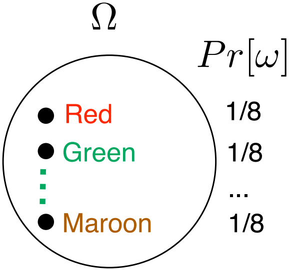

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$\Omega = \{$white, red, yellow, grey, purple, blue, maroon, green$\}$

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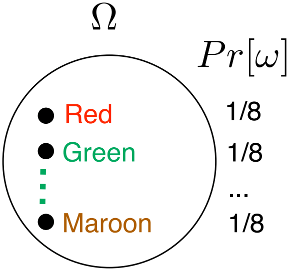

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A bag of identical balls, except for their color (or a label). If the bag is well shaken, every ball is equally likely to be picked.

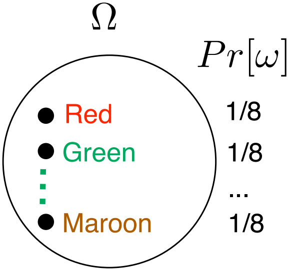
$$\Omega = \{\text{white, red, yellow, grey, purple, blue, maroon, green}\}$$

$$Pr[\text{blue}] =$$

Probability Space: Formalism

Simplest physical model of a uniform probability space:

Physical experiment



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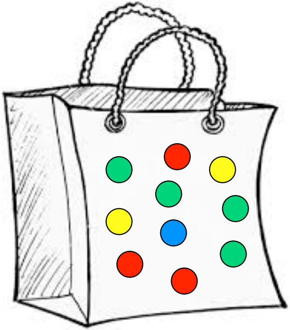

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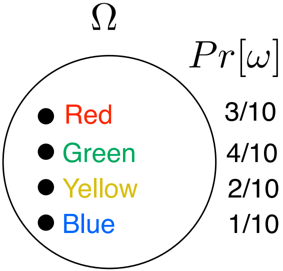

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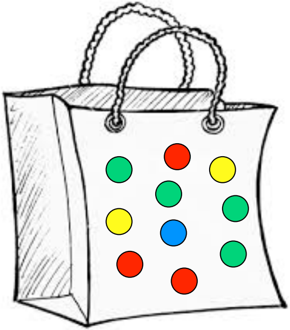

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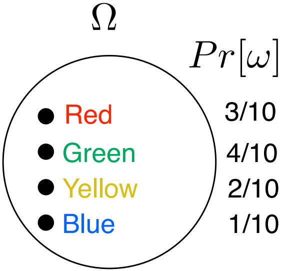

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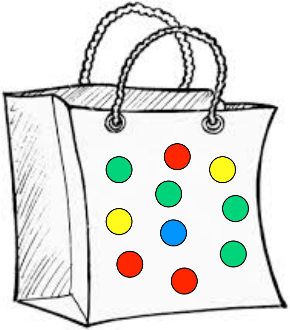

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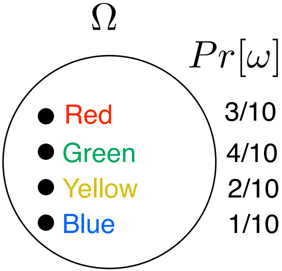

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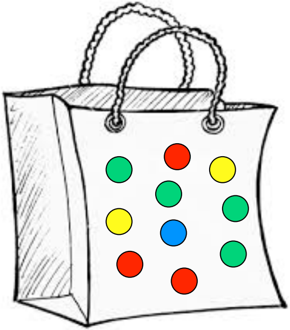

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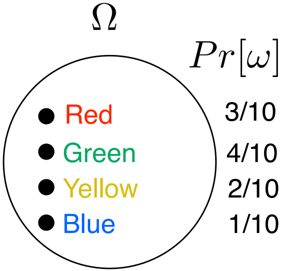

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$$\Omega = \{\text{Red, Green, Yellow, Blue}\}$$
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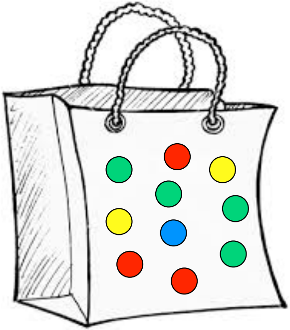

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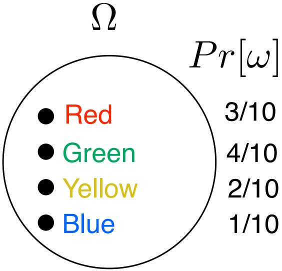

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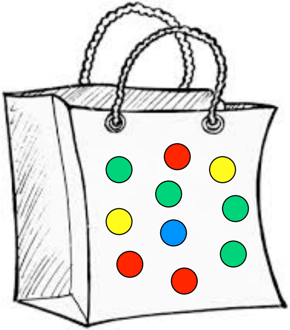

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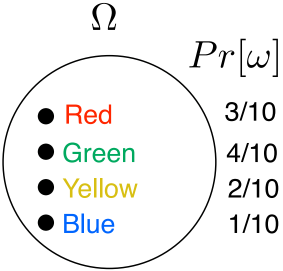

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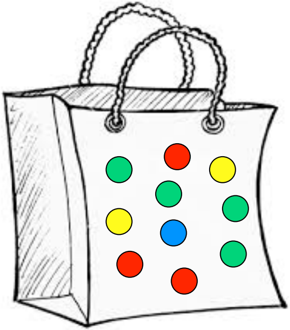

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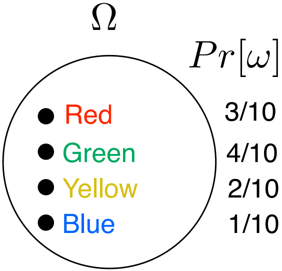

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Note: Probabilities are restricted to rational numbers: $\frac{N_k}{N}$.

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Physical model of a general non-uniform probability space:

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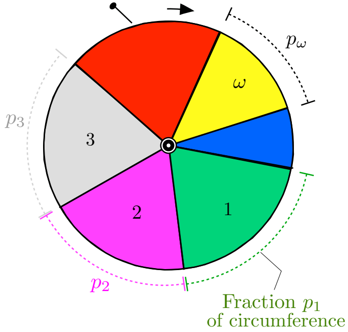

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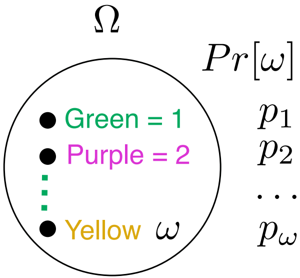

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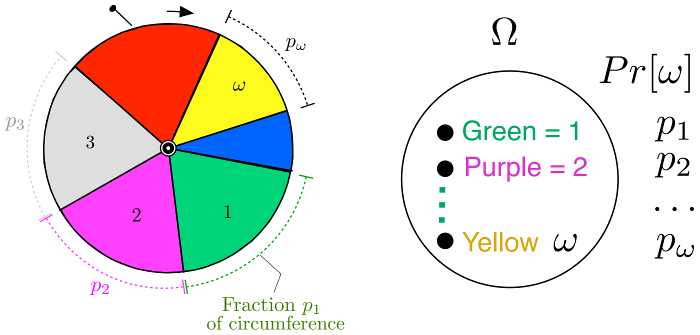

Physical experiment   Probability model

The roulette wheel stops in sector ω with probability p_ω.

Probability Space: Formalism

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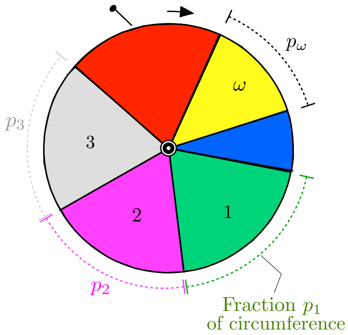

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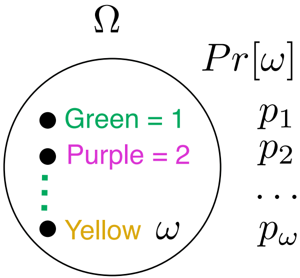

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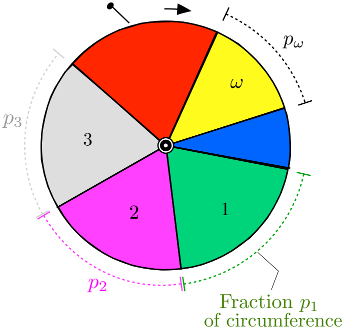

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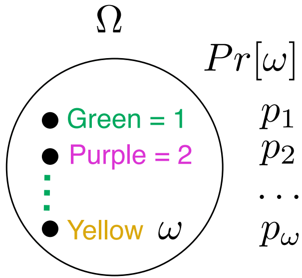

Probability model

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- For instance, say we glue the coins side-by-side so that they face up the same way. Then one gets *HH* or *TT* with probability 50% each. This is not captured by ‘picking two outcomes.’

Example: Monty Hall Problem

Game Show: 3 doors.

1. Awesome prize behind one of the doors.
2. Contestant picks a door. Monty opens **one of the other** two doors that does not have a prize.
3. Contestant can either stick to the original door or change selection.

Is there any advantage to the contestant in switching?

Controversy

The problem appeared in "Parade" Magazine in a column by Marilyn Vos Savant.

From the Wikipedia

Though vos Savant gave the correct answer that switching would win two-thirds of the time, she estimates the magazine received 10,000 letters including close to 1,000 signed by PhDs, many on letterheads of mathematics and science departments, declaring that her solution was wrong (Tierney 1991). As a result of the publicity the problem earned the alternative name Marilyn and the Goats.

Monty Hall problem

- Two experiments, one in which he switches and one in which he doesn't.
- $\Omega_1=$ {(Correct,Switch,Lose),(Wrong1,Switch,Win),(Wrong2,Switch,Win)} Prob of Winning is $\frac{2}{3}$.Another way: He only loses if picks the right door.
- $\Omega_2=${(Correct,Stay,Win),(Wrong1,Stay,Lose),(Wrong2,Stay,Lose)}.P of Winning is $\frac{1}{3}$.He only wins if he picks the right door.
- Much better to switch!

Why were so many professional mathematicians confused? They forgot about wrong door 2.

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Probability problems can be fun

.

The n passengers of a full flight board sequentially. The first passenger sits in the wrong (unassigned) spot. Subsequent passengers either sit in their assigned seat or if it is not available, pick another seat at random. What is the the probability that the last passenger is able to sit in his assigned seat?