CS70: Discrete Math and Probability.

First (half) week...

CS70: Discrete Math and Probability.

First (half) week...

Almost Done!

CS70: Discrete Math and Probability.

First (half) week...

Almost Done! Yaay!

CS70: Discrete Math and Probability.

First (half) week...

Almost Done! Yaay!

I hope you are getting into the flow.

CS70: Discrete Math and Probability.

First (half) week...

Almost Done! Yaay!

I hope you are getting into the flow.

Waitlist/concurrent enrollment.

CS70: Discrete Math and Probability.

First (half) week...

Almost Done! Yaay!

I hope you are getting into the flow.

Waitlist/concurrent enrollment.

Waitlist: in the past have gotten people in.

CS70: Discrete Math and Probability.

First (half) week...

Almost Done! Yaay!

I hope you are getting into the flow.

Waitlist/concurrent enrollment.

Waitlist: in the past have gotten people in.
Can't promise.

CS70: Discrete Math and Probability.

First (half) week...

Almost Done! Yaay!

I hope you are getting into the flow.

Waitlist/concurrent enrollment.

Waitlist: in the past have gotten people in.
Can't promise.

Concurrent Enrollment: not always accomodated.

CS70: Discrete Math and Probability.

First (half) week...

Almost Done! Yaay!

I hope you are getting into the flow.

Waitlist/concurrent enrollment.

Waitlist: in the past have gotten people in.
Can't promise.

Concurrent Enrollment: not always accomodated.
New scheme this year makes it easier.

CS70: Discrete Math and Probability.

First (half) week...

Almost Done! Yaay!

I hope you are getting into the flow.

Waitlist/concurrent enrollment.

Waitlist: in the past have gotten people in.
Can't promise.

Concurrent Enrollment: not always accomodated.
New scheme this year makes it easier.

Keep up, send email to fa17@eecs.org to get enrolled in gradescope, etc.

Last Time: The Language of Proofs.

Propositions:

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q$

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv$

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P$

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv$

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

$P(x) =$ '$x > 2$.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \lor Q$, $P \land Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \lor Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

$P(x) =$ '$x > 2$.

Quantifiers:

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

$P(x) =$ '$x > 2$.

Quantifiers:

$\forall x \in \mathbb{N}, x > 2$.

$\exists x \in \mathbb{N}, x > 2$.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

$P(x) =$ '$x > 2$.

Quantifiers:

$\forall x \in \mathbb{N}, x > 2$.

$\exists x \in \mathbb{N}, x > 2$.

Universe:

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

$P(x) =$ '$x > 2$.

Quantifiers:

$\forall x \in \mathbb{N}, x > 2$.

$\exists x \in \mathbb{N}, x > 2$.

Universe:

The milky way.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.
$3 > 2$.

Propositional Forms.
$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.
$\neg P \vee Q \equiv P \implies Q$.
$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:
Statements with free variables whose values determine truth.
$P(x) =$ '$x > 2$.

Quantifiers:
$\forall x \in \mathbb{N}, x > 2$.
$\exists x \in \mathbb{N}, x > 2$.

Universe:
The milky way. Kidding.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

$P(x) =$ '$x > 2$.

Quantifiers:

$\forall x \in \mathbb{N}, x > 2$.

$\exists x \in \mathbb{N}, x > 2$.

Universe:

The milky way. Kidding. Just trying to keep everyone awake.

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

$P(x) = 'x > 2$.

Quantifiers:

$\forall x \in \mathbb{N}, x > 2$.

$\exists x \in \mathbb{N}, x > 2$.

Universe:

The milky way. Kidding. Just trying to keep everyone awake.

$\mathbb{N}$,

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

$P(x) =$ '$x > 2$.

Quantifiers:

$\forall x \in \mathbb{N}, x > 2$.

$\exists x \in \mathbb{N}, x > 2$.

Universe:

The milky way. Kidding. Just trying to keep everyone awake.

$\mathbb{N}, \mathbb{Z}$,

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

$P(x) =$ '$x > 2$.

Quantifiers:

$\forall x \in \mathbb{N}, x > 2$.

$\exists x \in \mathbb{N}, x > 2$.

Universe:

The milky way. Kidding. Just trying to keep everyone awake.

$\mathbb{N}, \mathbb{Z}, \mathbb{R},$

Last Time: The Language of Proofs.

Propositions: Statements that are true or false.

$3 > 2$.

Propositional Forms.

$P \vee Q$, $P \wedge Q$, $\neg P$, $P \implies Q$

Truth Tables/Logical Equivalence.

$\neg P \vee Q \equiv P \implies Q$.

$\neg Q \implies \neg P \equiv P \implies Q$.

Predicates:

Statements with free variables whose values determine truth.

$P(x) =$ '$x > 2$.

Quantifiers:

$\forall x \in \mathbb{N}, x > 2$.

$\exists x \in \mathbb{N}, x > 2$.

Universe:

The milky way.   Kidding. Just trying to keep everyone awake.

$\mathbb{N}, \mathbb{Z}, \mathbb{R}, \ldots$

Back to: Wason's experiment:1

Theory:

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago."

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x)$

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B)$

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

$P(C)$ = True .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

$P(C)$ = True . Do we care about $Q(C)$?

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

$P(C)$ = True . Do we care about $Q(C)$?
Yes.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

$P(C)$ = True . Do we care about $Q(C)$?
Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

$P(C)$ = True . Do we care about $Q(C)$?
Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

$Q(D)$ = True .

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

$P(C)$ = True . Do we care about $Q(C)$?
Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

$Q(D)$ = True . Do we care about $P(D)$?

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

$P(C)$ = True . Do we care about $Q(C)$?
Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

$Q(D)$ = True . Do we care about $P(D)$?
No.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

$P(C)$ = True . Do we care about $Q(C)$?
Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

$Q(D)$ = True . Do we care about $P(D)$?
No. $P(D) \implies Q(D)$ holds whatever $P(D)$ is when $Q(D)$ is true.

Back to: Wason's experiment:1

Theory: "If a person travels to Chicago, he/she flies."

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

$P(x)$ = "Person x went to Chicago." $Q(x)$ = "Person x flew"

Statement/theory: $\forall x \in \{A, B, C, D\}, P(x) \implies Q(x)$

$P(A)$ = False . Do we care about $Q(A)$?
No. $P(A) \implies Q(A)$, when $P(A)$ is False , $Q(A)$ can be anything.

$Q(B)$ = False . Do we care about $P(B)$?
Yes. $P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B)$.
So $P(Bob)$ must be False .

$P(C)$ = True . Do we care about $Q(C)$?
Yes. $P(C) \implies Q(C)$ means $Q(C)$ must be true.

$Q(D)$ = True . Do we care about $P(D)$?
No. $P(D) \implies Q(D)$ holds whatever $P(D)$ is when $Q(D)$ is true.

Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

More for all quantifiers examples.

- “doubling a number always makes it larger”

More for all quantifiers examples.

- "doubling a number always makes it larger"

$$(\forall x \in N)\ (2x > x)$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

$$(\forall x \in N)\,(2x > x) \quad \textbf{False}$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

$$(\forall x \in N)\ (2x > x) \quad \textbf{False} \quad \textbf{Consider } x = 0$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $(\forall x \in N)$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $(\forall x \in N)(x > 5$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $(\forall x \in N)(x > 5 \implies$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $$(\forall x \in N)\ (2x > x) \quad \textbf{False} \quad \textbf{Consider } x = 0$$

  Can fix statement...

  $$(\forall x \in N)\ (2x \geq x) \quad \textbf{True}$$

- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

  Idea alert:

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $$(\forall x \in N)\ (2x > x) \quad \textbf{False} \quad \textbf{Consider } x = 0$$

  Can fix statement...

  $$(\forall x \in N)\ (2x \geq x) \quad \textbf{True}$$

- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

  Idea alert: Restrict domain using implication.

More for all quantifiers examples.

- "doubling a number always makes it larger"

  $(\forall x \in N)\ (2x > x)$ **False** **Consider** $x = 0$

  Can fix statement...

  $(\forall x \in N)\ (2x \geq x)$ **True**

- "Square of any natural number greater than 5 is greater than 25."

  $$(\forall x \in N)(x > 5 \implies x^2 > 25).$$

  Idea alert: Restrict domain using implication.

Note that we may omit universe if clear from context.

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)$$

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in N)\ (\forall x \in N)$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2)$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \qquad \text{False}$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \quad \text{False}$$

- In English: "the square of every natural number is a natural number."

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \quad \text{False}$$

- In English: "the square of every natural number is a natural number."

$$(\forall x \in N)$$

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \quad \text{False}$$

- In English: “the square of every natural number is a natural number.”

$$(\forall x \in N)(\exists y \in N)$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \qquad \text{False}$$

- In English: "the square of every natural number is a natural number."

$$(\forall x \in N)(\exists y \in N)\ (y = x^2)$$

Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \quad \text{False}$$

- In English: “the square of every natural number is a natural number.”

$$(\forall x \in N)(\exists y \in N)\ (y = x^2) \quad \text{True}$$

Quantifiers..not commutative.

- In English: "there is a natural number that is the square of every natural number".

$$(\exists y \in N)\ (\forall x \in N)\ (y = x^2) \quad \text{False}$$

- In English: "the square of every natural number is a natural number."

$$(\forall x \in N)(\exists y \in N)\ (y = x^2) \quad \text{True}$$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ “For all inputs x the program works.”

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
Counterexample.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
  Counterexample.
  Bad input.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
Counterexample.
Bad input.
Case that illustrates bug.

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
Counterexample.
Bad input.
Case that illustrates bug.
For True :

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
Counterexample.
Bad input.
Case that illustrates bug.
For True : prove claim!

Quantifiers....negation...DeMorgan again.

Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an x in S where $P(x)$ does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

Claim: $(\forall x)\ P(x)$ "For all inputs x the program works."
For False , find x, where $\neg P(x)$.
Counterexample.
Bad input.
Case that illustrates bug.
For True : prove claim! What we do in this course!

Negation of exists.

Consider

Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that for all x in S , $P(x)$ does not hold.

Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that for all x in S , $P(x)$ does not hold.

That is,

Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that for all x in S , $P(x)$ does not hold.

That is,

$$\neg(\exists x \in S)(P(x)) \iff \forall(x \in S)\neg P(x).$$

Which Theorem?

Theorem: $(\forall n \in N)\, \neg((\exists a, b, c \in N)\, (n \geq 3 \implies a^n + b^n = c^n))$

Which Theorem?

Theorem: $(\forall n \in N) \neg((\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n))$

Which Theorem?

Which Theorem?

Theorem: $(\forall n \in N) \neg((\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n))$

Which Theorem?

Fermat's Last Theorem!

Which Theorem?

Theorem: $(\forall n \in N)\, \neg((\exists a, b, c \in N)\, (n \geq 3 \implies a^n + b^n = c^n))$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles:

Which Theorem?

Theorem: $(\forall n \in N)\, \neg((\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n))$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles:
for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

Which Theorem?

Theorem: $(\forall n \in N) \neg((\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n))$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles:
for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn't fit in the margins.

Which Theorem?

Theorem: $(\forall n \in N)\, \neg((\exists a, b, c \in N)\, (n \geq 3 \implies a^n + b^n = c^n))$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles:
for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn't fit in the margins.

1993: Wiles ...(based in part on Ribet's Theorem)

Which Theorem?

Theorem: $(\forall n \in N) \neg((\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n))$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles:
for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn't fit in the margins.

1993: Wiles ...(based in part on Ribet's Theorem)

Movie – "Nova: The Proof."

Which Theorem?

Theorem: $(\forall n \in N) \neg((\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n))$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles:
for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn't fit in the margins.

1993: Wiles ...(based in part on Ribet's Theorem)

Movie – "Nova: The Proof."

DeMorgan Restatement:

Which Theorem?

Theorem: $(\forall n \in N)\ \neg((\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n))$

Which Theorem?

Fermat's Last Theorem!

Remember Special Triangles:
  for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

1637: Proof doesn't fit in the margins.

1993: Wiles ...(based in part on Ribet's Theorem)

Movie – "Nova: The Proof."

DeMorgan Restatement:
Theorem: $\neg(\exists n \in N)\ (\exists a, b, c \in N)\ (n \geq 3 \implies a^n + b^n = c^n)$

Summary.

Propositions are statements that are true or false.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with "free" variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems!

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with "free" variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: "Flip and Distribute negation"

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: “Flip and Distribute negation”
$\neg(P \vee Q) \iff$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: “Flip and Distribute negation”
$\neg(P \vee Q) \iff (\neg P \wedge \neg Q)$
$\neg \forall x\ P(x) \iff$

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with "free" variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: "Flip and Distribute negation"
$\neg(P \vee Q) \iff (\neg P \wedge \neg Q)$
$\neg \forall x\ P(x) \iff \quad \exists x\ \neg P(x)$.

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: “Flip and Distribute negation”

$\neg(P \vee Q) \iff (\neg P \wedge \neg Q)$

$\neg \forall x\ P(x) \iff \quad \exists x\ \neg P(x)$.

A bit dry...

Summary.

Propositions are statements that are true or false.

Proprositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with "free" variables.

Quantifiers: $\forall x\ P(x)$, $\exists y\ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgans Laws: "Flip and Distribute negation"
$\neg(P \vee Q) \iff (\neg P \wedge \neg Q)$
$\neg \forall x\ P(x) \iff \exists x\ \neg P(x)$.

A bit dry...

Why?

From the langauge of Proofs...

From the langauge of Proofs...

to

From the langauge of Proofs...

to
Proofs.

Yaay!

And now:

Yaay!

And now: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

Quick Background and Notation.

Integers closed under addition.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means “a divides b”.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a+b \in Z$

$a|b$ means "a divides b".

$2|4$?

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means “a divides b”.

$2|4$? Yes!

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$?

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means “a divides b”.

$2|4$? Yes!

$7|23$? No!

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means “a divides b”.

$2|4$? Yes!

$7|23$? No!

$4|2$?

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means “a divides b”.

$2|4$? Yes!

$7|23$? No!

$4|2$? No!

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

2|4? Yes!

7|23? No!

4|2? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$,

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes!

$7|23$? No!

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$, $15 = 3(5)$.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes! Since for $q = 2$, $4 = (2)2$.

$7|23$? No!

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$, $15 = 3(5)$.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes! Since for $q = 2$, $4 = (2)2$.

$7|23$? No! No q where true.

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$, $15 = 3(5)$.

Quick Background and Notation.

Integers closed under addition.

$a, b \in Z \implies a + b \in Z$

$a|b$ means "a divides b".

$2|4$? Yes! Since for $q = 2$, $4 = (2)2$.

$7|23$? No! No q where true.

$4|2$? No!

Formally: $a|b \iff \exists q \in Z$ where $b = aq$.

$3|15$ since for $q = 5$, $15 = 3(5)$.

A natural number $p > 1$, is **prime** if it is divisible only by 1 and itself.

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$

Direct Proof.

Theorem: For any $a,b,c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$

$b = aq$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq'$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$ and $(q - q')$ is an integer so

$a|(b - c)$

Direct Proof.

Theorem: For any $a,b,c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q,q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

$a|(b-c)$ □

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

$a|(b-c)$ $\square$

Works for $\forall a, b, c$?

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

$a|(b-c)$ $\square$

Works for $\forall a, b, c$?
Argument applies to *every* $a, b, c \in Z$.

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$ and $(q - q')$ is an integer so

$a|(b - c)$ □

Works for $\forall a, b, c$?
Argument applies to *every* $a, b, c \in Z$.

Direct Proof Form:

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$ and $(q - q')$ is an integer so

$a|(b - c)$ $\square$

Works for $\forall a, b, c$?
Argument applies to *every* $a, b, c \in Z$.

Direct Proof Form:
Goal: $P \implies Q$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b-c) = a(q-q')$ and $(q-q')$ is an integer so

$a|(b-c)$ $\square$

Works for $\forall a, b, c$?
Argument applies to *every* $a, b, c \in Z$.

Direct Proof Form:
Goal: $P \implies Q$
Assume P.

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$ and $(q - q')$ is an integer so

$a|(b - c)$ $\square$

Works for $\forall a, b, c$?
Argument applies to *every* $a, b, c \in Z$.

Direct Proof Form:
Goal: $P \implies Q$
Assume P.
$\ldots$

Direct Proof.

Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

Proof: Assume $a|b$ and $a|c$
$b = aq$ and $c = aq'$ where $q, q' \in Z$

$b - c = aq - aq' = a(q - q')$ Done?

$(b - c) = a(q - q')$ and $(q - q')$ is an integer so

$a|(b-c)$ $\square$

Works for $\forall a, b, c$?
Argument applies to *every* $a, b, c \in Z$.

Direct Proof Form:
Goal: $P \implies Q$
Assume P.
...
Therefore Q.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:
$n = 121$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:
$n = 121$ Alt Sum: $1 - 2 + 1 = 0$.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:
$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:
$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is 605

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$,

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum:

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1-2+1=0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6-0+5=11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b$$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n,

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1-2+1=0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6-0+5=11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a+10b+c$, for some a, b, c.

Assume: Alt. sum: $a-b+c = 11k$ for some integer k.

Add $99a+11b$ to both sides.

$$100a+10b+c = 11k+99a+11b = 11(k+9a+b)$$

Left hand side is n, $k+9a+b$ is integer. $\implies 11|n$.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer. $\implies 11|n$. □

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer. $\implies 11|n$. □

Direct proof of $P \implies Q$:
Assumed P: $11|a - b + c$.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

Theorem: For $n \in D_3$, if the alternating sum of digits of n is divisible by 11, than $11|n$.

$$\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$$

Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

Assume: Alt. sum: $a - b + c = 11k$ for some integer k.

Add $99a + 11b$ to both sides.

$$100a + 10b + c = 11k + 99a + 11b = 11(k + 9a + b)$$

Left hand side is n, $k + 9a + b$ is integer. $\implies 11|n$. □

Direct proof of $P \implies Q$:
Assumed P: $11|a - b + c$. Proved Q: $11|n$.

The Converse

Thm: $\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$

The Converse

Thm: $\forall n \in D_3, (11 | \text{alt. sum of digits of } n) \implies 11 | n$

Is converse a theorem?

$\forall n \in D_3, (11 | n) \implies (11 | \text{alt. sum of digits of } n)$

The Converse

Thm: $\forall n \in D_3, (11|\text{alt. sum of digits of } n) \implies 11|n$

Is converse a theorem?
$\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$

Yes?

The Converse

Thm: $\forall n \in D_3, (11 | \text{alt. sum of digits of } n) \implies 11 | n$

Is converse a theorem?
$\forall n \in D_3, (11 | n) \implies (11 | \text{alt. sum of digits of } n)$

Yes? No?

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof:

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$n = 100a + 10b + c = 11k$$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$n = 100a + 10b + c = 11k \implies$
$99a + 11b + (a - b + c) = 11k$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$

Proof: Assume $11|n$.

$$n = 100a + 10b + c = 11k \implies$$
$$99a + 11b + (a - b + c) = 11k \implies$$
$$a - b + c = 11k - 99a - 11b$$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$

Proof: Assume $11|n$.

$$
\begin{aligned}
n = 100a + 10b + c = 11k &\implies \\
99a + 11b + (a - b + c) = 11k &\implies \\
a - b + c = 11k - 99a - 11b &\implies \\
a - b + c = 11(k - 9a - b)&
\end{aligned}
$$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$
\begin{aligned}
n = 100a + 10b + c = 11k &\implies \\
99a + 11b + (a - b + c) = 11k &\implies \\
a - b + c = 11k - 99a - 11b &\implies \\
a - b + c = 11(k - 9a - b) &\implies \\
a - b + c = 11\ell &
\end{aligned}
$$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$n = 100a + 10b + c = 11k \implies$$
$$99a + 11b + (a - b + c) = 11k \implies$$
$$a - b + c = 11k - 99a - 11b \implies$$
$$a - b + c = 11(k - 9a - b) \implies$$
$$a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z$$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$\begin{aligned} n = 100a + 10b + c = 11k &\implies \\ 99a + 11b + (a - b + c) = 11k &\implies \\ a - b + c = 11k - 99a - 11b &\implies \\ a - b + c = 11(k - 9a - b) &\implies \\ a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) &\in Z \end{aligned}$$

That is 11|alternating sum of digits. □

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$
\begin{aligned}
n = 100a + 10b + c = 11k &\implies \\
99a + 11b + (a - b + c) = 11k &\implies \\
a - b + c = 11k - 99a - 11b &\implies \\
a - b + c = 11(k - 9a - b) &\implies \\
a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) &\in Z
\end{aligned}
$$

That is 11|alternating sum of digits. □

Note: similar proof to other. In this case every $\implies$ is $\iff$

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$
\begin{aligned}
n = 100a + 10b + c = 11k \implies \\
99a + 11b + (a - b + c) = 11k \implies \\
a - b + c = 11k - 99a - 11b \implies \\
a - b + c = 11(k - 9a - b) \implies \\
a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z
\end{aligned}
$$

That is $11|$alternating sum of digits. □

Note: similar proof to other. In this case every $\implies$ is $\iff$

Often works with arithmetic properties ...

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$

Proof: Assume $11|n$.

$$n = 100a + 10b + c = 11k \implies$$
$$99a + 11b + (a - b + c) = 11k \implies$$
$$a - b + c = 11k - 99a - 11b \implies$$
$$a - b + c = 11(k - 9a - b) \implies$$
$$a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z$$

That is 11|alternating sum of digits. □

Note: similar proof to other. In this case every $\implies$ is $\iff$

Often works with arithmetic properties ...
...not when multiplying by 0.

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$n = 100a + 10b + c = 11k \implies$$
$$99a + 11b + (a - b + c) = 11k \implies$$
$$a - b + c = 11k - 99a - 11b \implies$$
$$a - b + c = 11(k - 9a - b) \implies$$
$$a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z$$

That is $11|$alternating sum of digits. □

Note: similar proof to other. In this case every $\implies$ is $\iff$

Often works with arithmetic properties ...
...not when multiplying by 0.

We have.

Another Direct Proof.

Theorem: $\forall n \in D_3, (11|n) \implies (11|\text{alt. sum of digits of } n)$
Proof: Assume $11|n$.

$$\begin{aligned} n = 100a + 10b + c = 11k &\implies \\ 99a + 11b + (a - b + c) = 11k &\implies \\ a - b + c = 11k - 99a - 11b &\implies \\ a - b + c = 11(k - 9a - b) &\implies \\ a - b + c = 11\ell \text{ where } \ell = (k - 9a - b) \in Z & \end{aligned}$$

That is 11|alternating sum of digits. □

Note: similar proof to other. In this case every $\implies$ is $\iff$

Often works with arithmetic properties ...
...not when multiplying by 0.

We have.

Theorem: $\forall n \in N', (11|\text{alt. sum of digits of } n) \iff (11|n)$

Proof by Contraposition

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do?

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd = q(2k)$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd = q(2k) = 2(kq)$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd = q(2k) = 2(kq)$

n is even.

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k+1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$$n = qd = q(2k) = 2(kq)$$

n is even. $\neg P$

Proof by Contraposition

Thm: For $n \in Z^+$ and $d|n$. If n is odd then d is odd.

$n = 2k + 1$ what do we know about d?

What to do? Is it even true?
Hey, that rhymes ...and there is a pun ... colored blue.
Anyway, what to do?

Goal: Prove $P \implies Q$.

Assume $\neg Q$
...and prove $\neg P$.

Conclusion: $\neg Q \implies \neg P$ equivalent to $P \implies Q$.

Proof: Assume $\neg Q$: d is even. $d = 2k$.

$d|n$ so we have

$n = qd = q(2k) = 2(kq)$

n is even. $\neg P$ $\square$

Another Contraposition...

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

n^2 is even, $n^2 = 2k$, ...

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

n^2 is even, $n^2 = 2k$, ...$\sqrt{2k}$ even?

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' ....

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' ....

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

... and n^2 is odd!

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

... and n^2 is odd!

$\neg Q \implies \neg P$

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

... and n^2 is odd!

$\neg Q \implies \neg P$ so $P \implies Q$ and ...

Another Contraposition...

Lemma: For every n in N, n^2 is even $\implies$ n is even. ($P \implies Q$)

Proof by contraposition: $(P \implies Q) \equiv (\neg Q \implies \neg P)$

$P =$ 'n^2 is even.' .... $\neg P =$ 'n^2 is odd'

$Q =$ 'n is even' .... $\neg Q =$ 'n is odd'

Prove $\neg Q \implies \neg P$: n is odd $\implies$ n^2 is odd.

$n = 2k + 1$

$n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1.$

$n^2 = 2l + 1$ where l is a natural number..

... and n^2 is odd!

$\neg Q \implies \neg P$ so $P \implies Q$ and ... $\square$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show:

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$,

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always “not” hold.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always “not” hold.

Proof by contradiction:

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always “not” hold.

Proof by contradiction:

Theorem: P.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always “not” hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always “not” hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always “not” hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R$

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R \equiv$ False

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R \equiv$ False

$\neg P \implies$ *False*

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R \equiv$ False

$\neg P \implies \mathit{False}$

Contrapositive:

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always “not” hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R \equiv$ False

$\neg P \implies$ *False*

Contrapositive: True $\implies P$.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R \equiv$ False

$\neg P \implies$ *False*

Contrapositive: True $\implies P$. Theorem P is proven.

Proof by contradiction:form

Theorem: $\sqrt{2}$ is irrational.

Must show: For every $a, b \in Z$, $(\frac{a}{b})^2 \neq 2$.

A simple property (equality) should always "not" hold.

Proof by contradiction:

Theorem: P.

$\neg P \implies P_1 \cdots \implies R$

$\neg P \implies Q_1 \cdots \implies \neg R$

$\neg P \implies R \wedge \neg R \equiv$ False

$\neg P \implies$ *False*

Contrapositive: True $\implies P$. Theorem P is proven. $\square$

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$:

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2$$

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2$$

a^2 is even $\implies$ a is even.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

$$b^2 = 2k^2$$

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

$$b^2 = 2k^2$$

b^2 is even $\implies$ b is even.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

$$b^2 = 2k^2$$

b^2 is even $\implies$ b is even.
a and b have a common factor.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

$$b^2 = 2k^2$$

b^2 is even $\implies$ b is even.
a and b have a common factor. Contradiction.

Contradiction

Theorem: $\sqrt{2}$ is irrational.

Assume $\neg P$: $\sqrt{2} = a/b$ for $a, b \in Z$.

Reduced form: a and b have no common factors.

$$\sqrt{2}b = a$$

$$2b^2 = a^2 = 4k^2$$

a^2 is even $\implies$ a is even.

$a = 2k$ for some integer k

$$b^2 = 2k^2$$

b^2 is even $\implies$ b is even.
a and b have a common factor. Contradiction.

□

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q,

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q$

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q$

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1$.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1$. or $p | 1$.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1$. or $p | 1$.
- so $p \leq 1$.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1.$ or $p | 1.$
- so $p \leq 1$. (**Contradicts *R*.**)

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1.$ or $p | 1$.
- so $p \leq 1$. (**Contradicts *R*.**)

The original assumption that "the theorem is false" is false, thus the theorem is proven.

Proof by contradiction: example

Theorem: There are infinitely many primes.

Proof:

- Assume finitely many primes: $p_1, \ldots, p_k$.
- Consider number

$$q = (p_1 \times p_2 \times \cdots p_k) + 1.$$

- q cannot be one of the primes as it is larger than any p_i.
- q has prime divisor p ("$p > 1$" = R) which is one of p_i.
- p divides both $x = p_1 \cdot p_2 \cdots p_k$ and q, and divides $x - q$,
- $\implies p | x - q \implies p \leq x - q = 1.$ or $p | 1.$
- so $p \leq 1$. (**Contradicts *R*.**)

The original assumption that "the theorem is false" is false, thus the theorem is proven. $\square$

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.
- The chain of reasoning started with a false statement.

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.
- The chain of reasoning started with a false statement.

Consider example..

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.
- The chain of reasoning started with a false statement.

Consider example..

- $2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031 = 59 \times 509$

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.
- The chain of reasoning started with a false statement.

Consider example..

- $2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031 = 59 \times 509$
- There is a prime *in between* 13 and $q = 30031$ that divides q.

Product of first k primes..

Did we prove?

- "The product of the first k primes plus 1 is prime."
- No.
- The chain of reasoning started with a false statement.

Consider example..

- $2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031 = 59 \times 509$
- There is a prime *in between* 13 and $q = 30031$ that divides q.
- Proof assumed no primes *in between* p_k and q.

And…

And...

Happy Friday!

And...

Happy Friday!

Enjoy your weekend...

And...

Happy Friday!

Enjoy your weekend...

...and take care.