Continuing Probability.

Wrap up: Total Probability and Conditional Probability.

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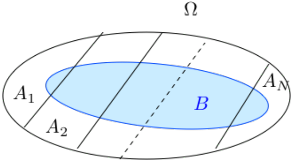
Product Rule, Correlation, Independence, Bayes' Rule,

Total probability

Assume that Ω is the union of the disjoint sets $A_1, \ldots, A_N$.



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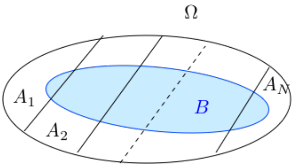

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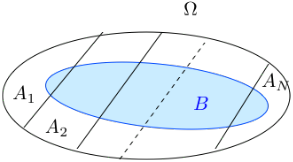

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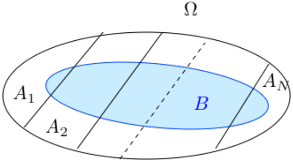

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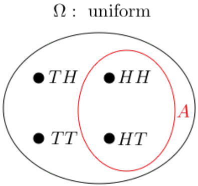

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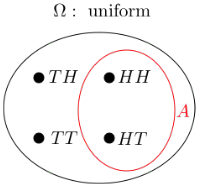

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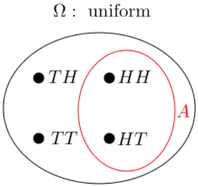

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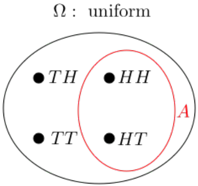

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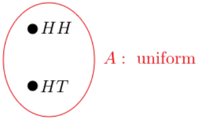

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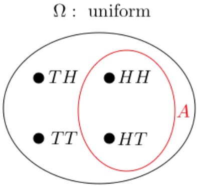

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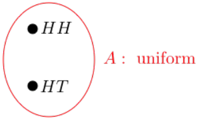

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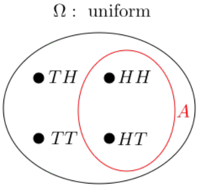

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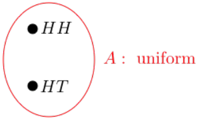

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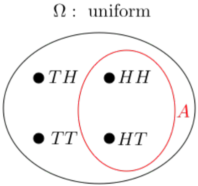

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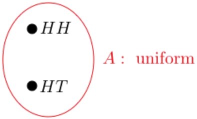

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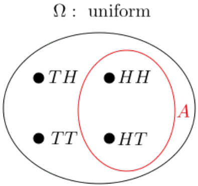

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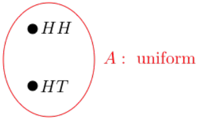

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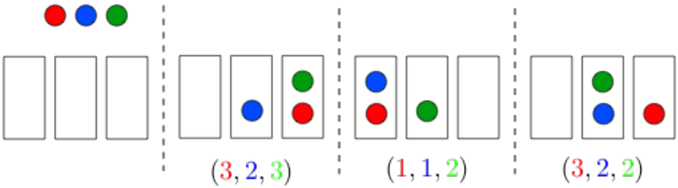

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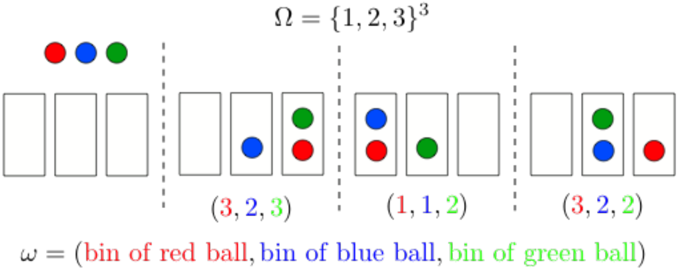

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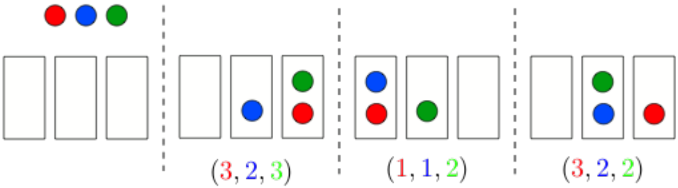

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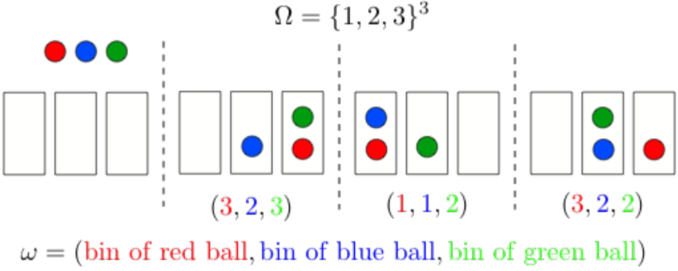

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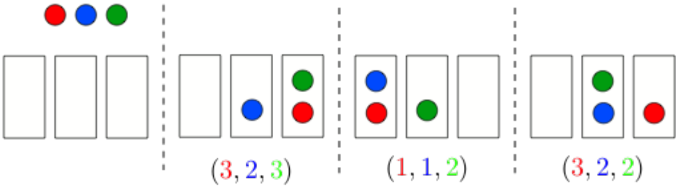

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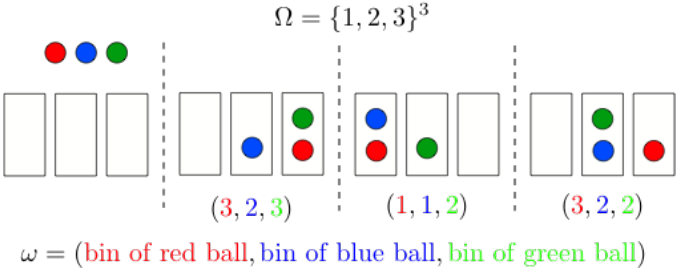

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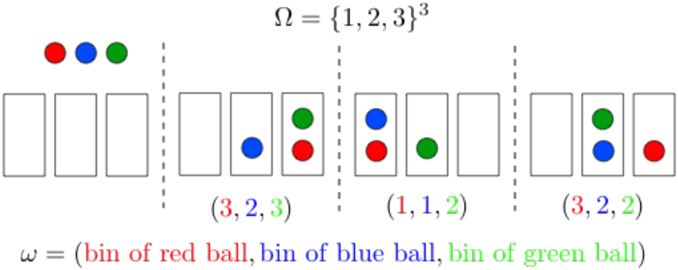

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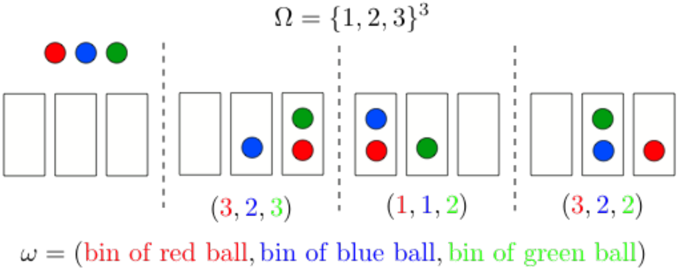

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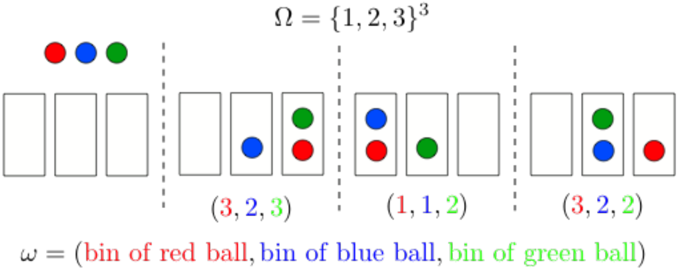

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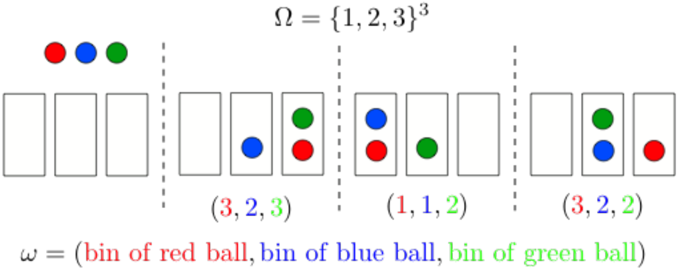

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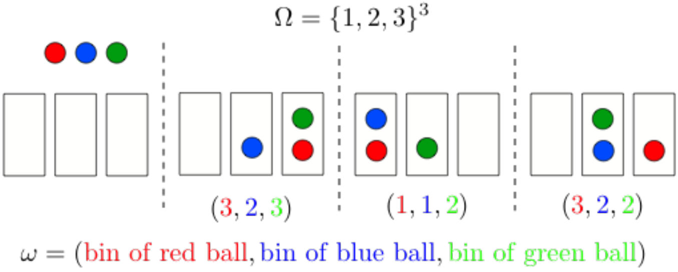

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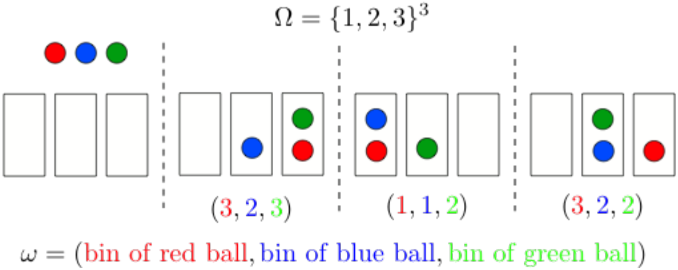

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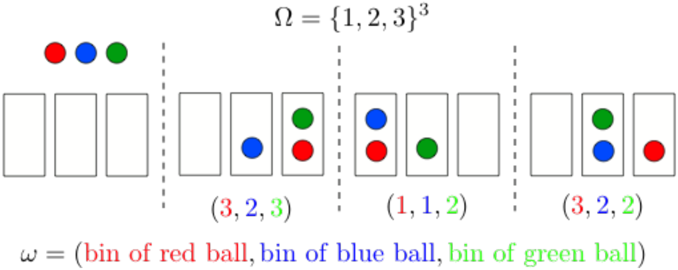

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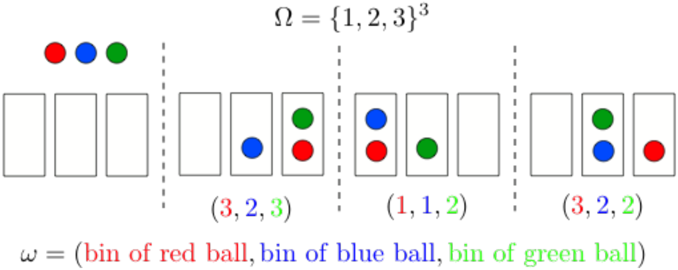

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A is less likely given B: If second bin is empty the first is more likely to have balls in it.

Three Card Problem

Three cards: Red/Red, Red/Black, Black/Black.
Pick one at random and place on the table. The upturned side is a Red. What is the probability that the other side is Black?
Can't be the BB card, so...prob should be 0.5, right?
R: upturned card is Red; RB: the Red/Black card was selected.
Want $P(RB|R)$.
What's wrong with the reasoning that leads to $\frac{1}{2}$?

$$\begin{aligned} P(RB|R) &= \frac{P(RB \cap R)}{P(R)} \\ &= \frac{\frac{1}{3}\frac{1}{2}}{\frac{1}{3}(1) + \frac{1}{3}\frac{1}{2} + \frac{1}{3}(0)} \\ &= \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3} \end{aligned}$$

Once you are given R: it is twice as likely that the RR card was picked.

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Flip a fair coin 51 times.

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The likelihood of 51st heads does not depend on the previous flips.

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Recall the definition:

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Let $A_1, A_2, \ldots, A_n$ be events. Then

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$$Pr[A_1 \cap \cdots \cap A_n] = Pr[A_1] Pr[A_2 | A_1] \cdots Pr[A_n | A_1 \cap \cdots \cap A_{n-1}].$$

Product Rule

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Let $A_1, A_2, \ldots, A_n$ be events. Then

$$Pr[A_1 \cap \cdots \cap A_n] = Pr[A_1] Pr[A_2 | A_1] \cdots Pr[A_n | A_1 \cap \cdots \cap A_{n-1}].$$

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so that the result holds for $n + 1$. □

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Random experiment: Pick a person at random.

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- Smoking increases the probability of lung cancer by 17%.

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Event A: the person has lung cancer. Event B: the person is a heavy smoker. $Pr[A|B] = 1.17 \times Pr[A]$.

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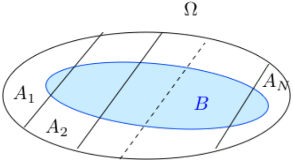
Total probability

Assume that Ω is the union of the disjoint sets $A_1, \ldots, A_N$.



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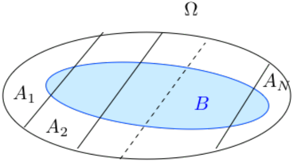

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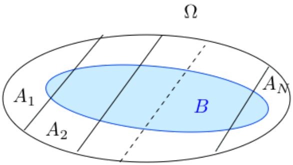

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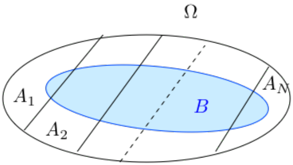

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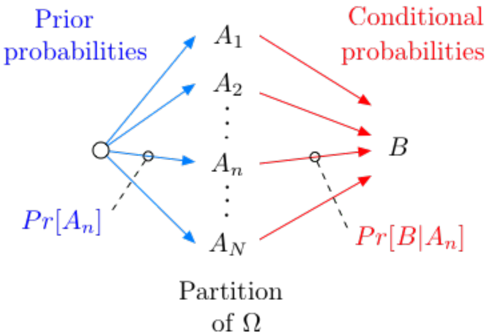

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Thus,

$$Pr[A|B] = \frac{Pr[A]Pr[B|A]}{Pr[B]} = \frac{(1/2)(1/2)}{(1/2)(1/2) + (1/2)0.6} \approx 0.45.$$

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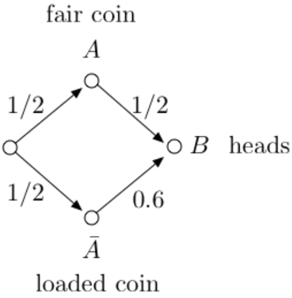

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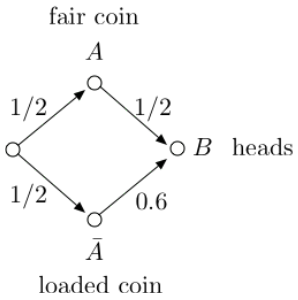

Imagine 100 situations, among which
$m := 100(1/2)(1/2)$ are such that A and B occur and
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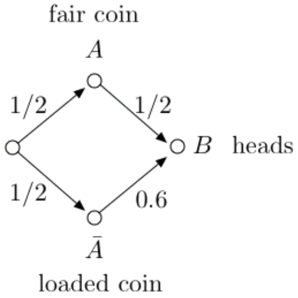

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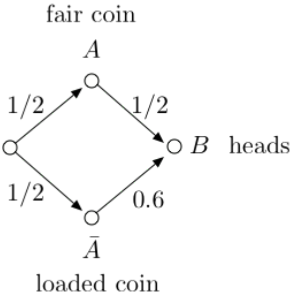

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$$Pr[A \cap B] = Pr[A] Pr[B].$$

Examples:

- When rolling two dice, $A =$ sum is 7 and $B =$ red die is 1 are independent;
- When rolling two dice, $A =$ sum is 3 and $B =$ red die is 1 are not independent;
- When flipping coins, $A =$ coin 1 yields heads and $B =$ coin 2 yields tails are independent;
- When throwing 3 balls into 3 bins, $A =$ bin 1 is empty and $B =$ bin 2 is empty are not independent;

Independence and conditional probability

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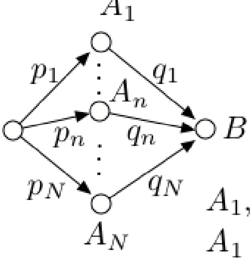

$$p_n = Pr[A_n]$$

$$q_n = Pr[B|A_n]$$

$A_1, \ldots, A_N$ disjoint

$A_1 \cup \cdots \cup A_N = \Omega$

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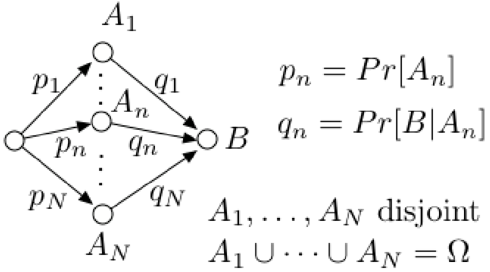

Imagine 100 situations, among which $100 p_n q_n$ are such that A_n and B occur, for $n = 1, \ldots, N$.

Thus, among the $100 \sum_m p_m q_m$ situations where B occurred, there are $100 p_n q_n$ where A_n occurred.

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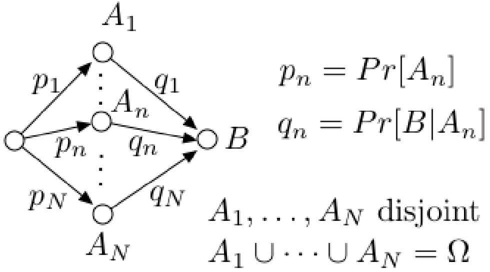

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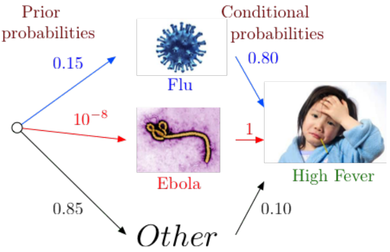
Hence,

$$Pr[A_n|B] = \frac{p_n q_n}{\sum_m p_m q_m}.$$

Why do you have a fever?



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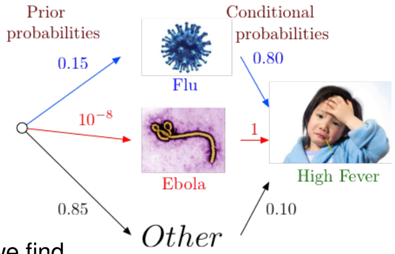

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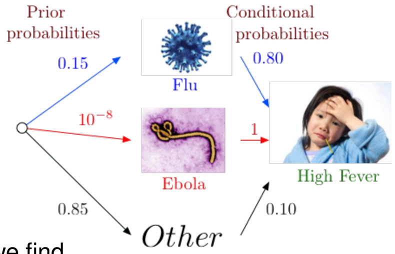

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$$Pr[\text{Flu}|\text{High Fever}] = \frac{0.15 \times 0.80}{0.15 \times 0.80 + 10^{-8} \times 1 + 0.85 \times 0.1} \approx 0.58$$

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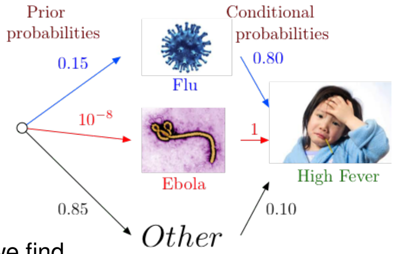

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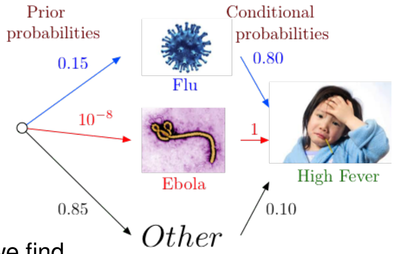

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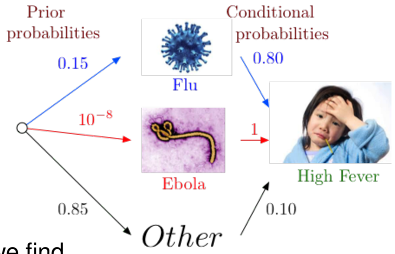

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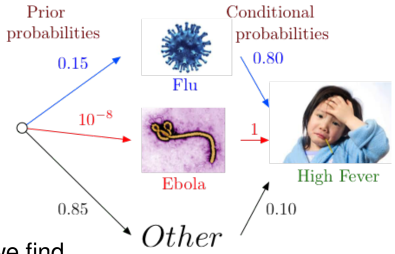

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Bayes’ Rule Operations

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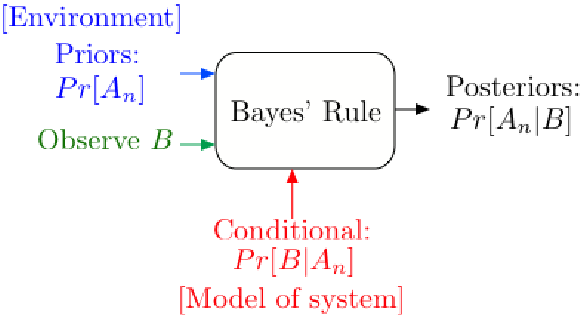

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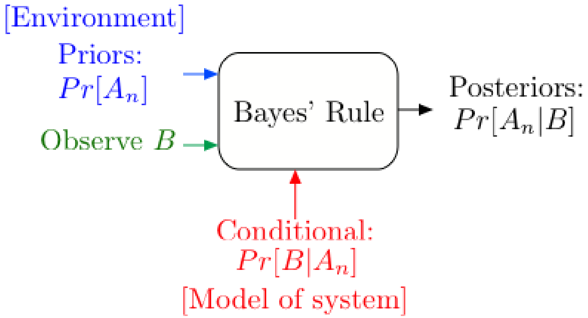

Bayes' Rule is the canonical example of how information changes our opinions.

Thomas Bayes

Thomas Bayes

Portrait used of Bayes in a 1936 book,[1] but it is doubtful whether the portrait is actually of him.[2] No earlier portrait or claimed portrait survives.

Born	c. 1701 London, England
Died	7 April 1761 (aged 59) Tunbridge Wells, Kent, England
Residence	Tunbridge Wells, Kent, England
Nationality	English
Known for	Bayes' theorem

Source: Wikipedia.

Thomas Bayes

A

FIG. 3. *Joshua Bayes (1671–1746).*

$Pr[B|A]$ →

Max over B

B

Thomas Bayes?

A Bayesian picture of Thomas Bayes.

Testing for disease.

Let’s watch TV!!

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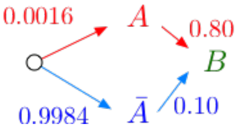

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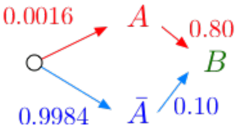

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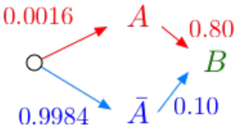

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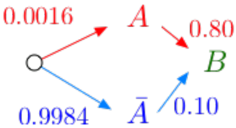

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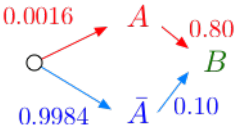

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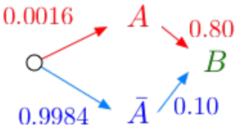

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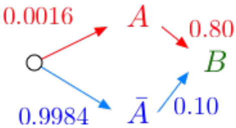

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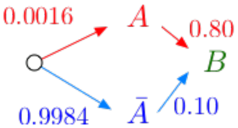

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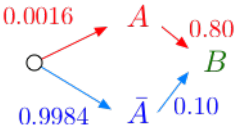

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