CS70: Random Variables (contd.)

Important Distributions

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Important Distributions

1. Expectation: Brief Review
2. Important Distributions: Binomial, Uniform, Geometric, Poisson

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- $E[aX + bY + c] = aE[X] + bE[Y] + c$.

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Note: If we had defined $Y = a_1X_1 + \cdots + a_nX_n$ has had tried to compute $E[Y] = \sum_y yPr[Y = y]$, we would have been in trouble!

Using Linearity: Binomial Distribution.

Flip n coins with heads probability p.

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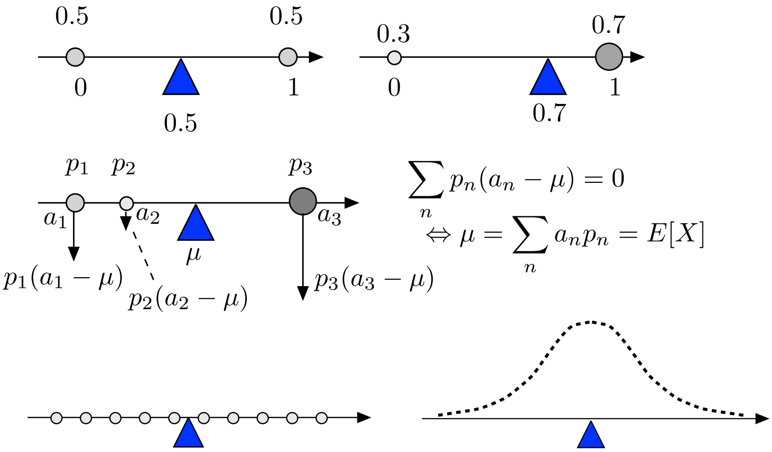

$$\sum_n p_n(a_n - \mu) = 0$$

$$\Leftrightarrow \mu = \sum_n a_n p_n = E[X]$$

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Thus, we will write $X = 1_A$.

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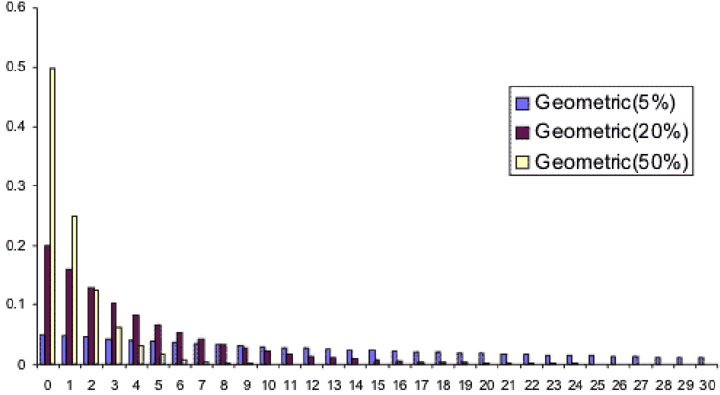

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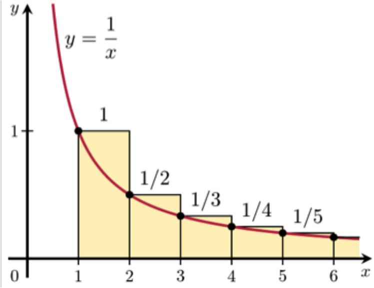

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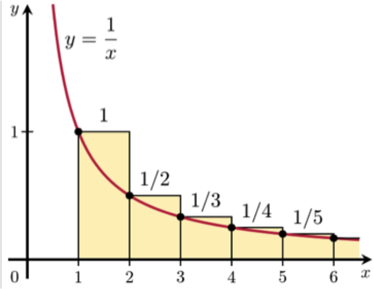

A good approximation is

$H(n) \approx \ln(n) + \gamma$ where $\gamma \approx 0.58$ (Euler-Mascheroni constant).

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Let X be $G(p)$. Then, for $n \geq 0$,

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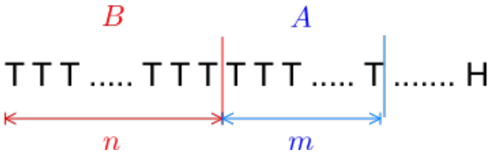

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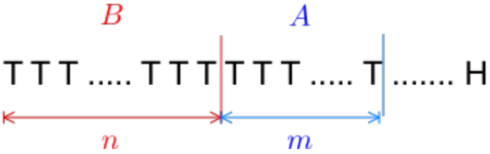

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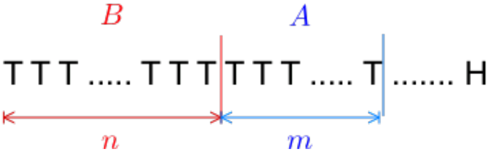

$$Pr[X > n+m | X > n] = Pr[A|B] = Pr[A] = Pr[X > m].$$

The coin is memoryless, therefore, so is X.

Geometric Distribution: Yet another look

Theorem: For a r.v. X that takes the values $\{0,1,2,\ldots\}$, one has

$$E[X] = \sum_{i=1}^{\infty} Pr[X \geq i].$$

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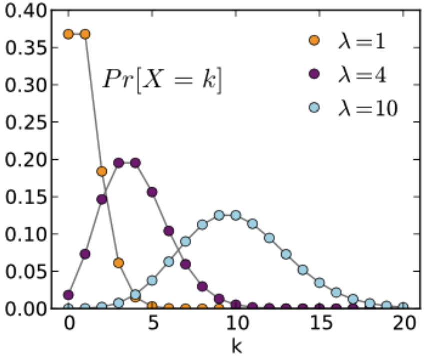

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For (1) we used $m \ll n$; for (2) we used $(1 - a/n) \approx e^{-a/n}$ for $a/n \ll 1$.

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