CS 70: Discrete Math and Probability.

More.

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Principle of Induction.

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$\forall n \in \mathbb{N} P(n)$

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Strenthening Induction Hypothesis.

Theorem: The sum of the first n odd numbers is a perfect square.

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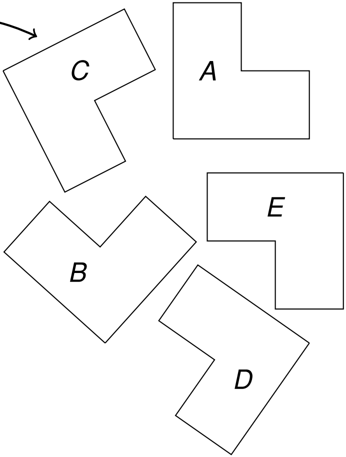
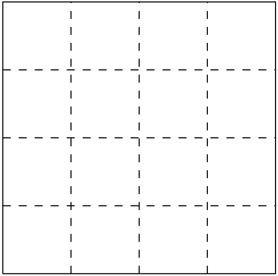
□

Tiling Cory Hall Courtyard.

Use these L-tiles.



To Tile this 4×4 courtyard.



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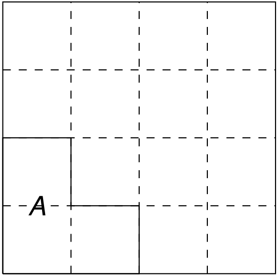

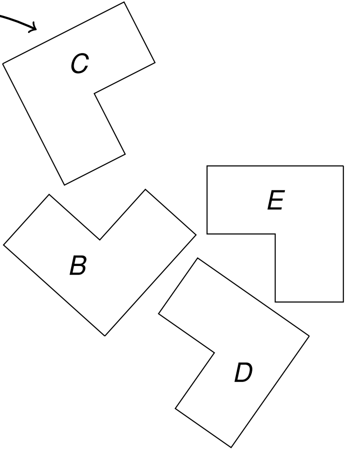

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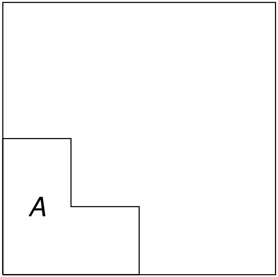

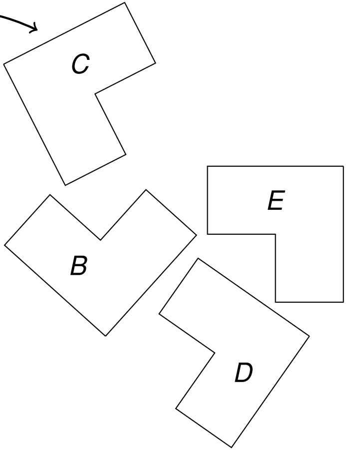

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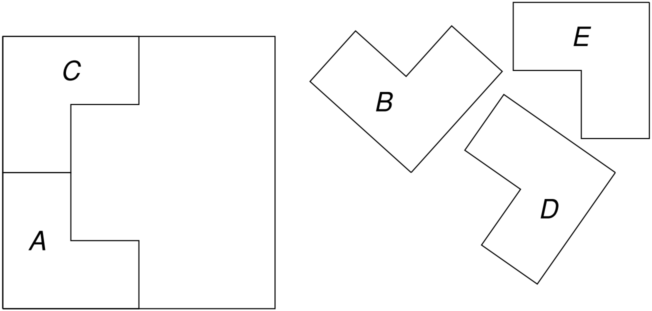

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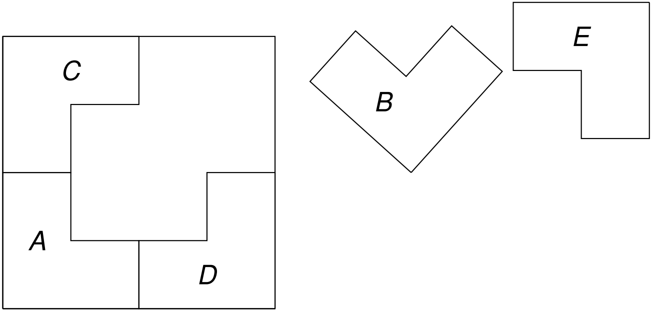

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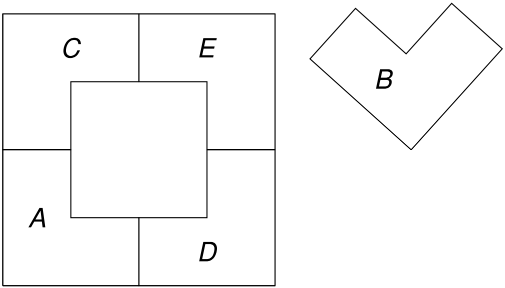

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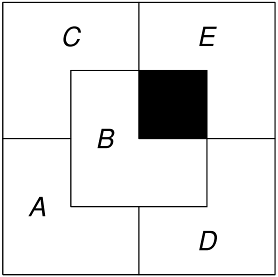

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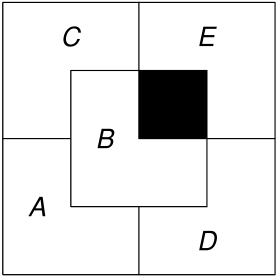

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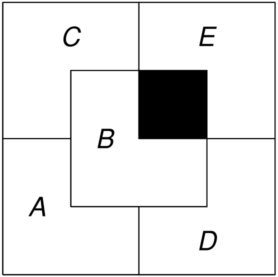


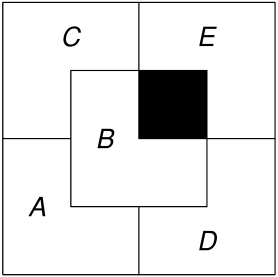
Alright!

Tiled 4×4 square with 2×2 L-tiles.

Tiling Cory Hall Courtyard.

Use these L-tiles.

To Tile this 4×4 courtyard.



Alright!

Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Tiling Cory Hall Courtyard.

Use these L-tiles.

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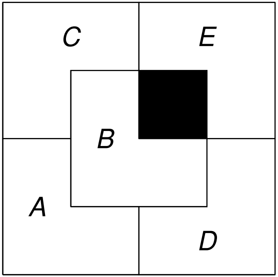

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Can we tile any $2^n \times 2^n$ with L-tiles (with a hole)

Tiling Cory Hall Courtyard.

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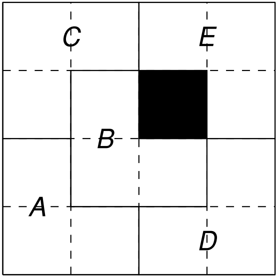

Alright!

Tiled 4×4 square with 2×2 L-tiles.

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Can we tile any $2^n \times 2^n$ with L-tiles (with a hole) for every n!

Hole have to be there? Maybe just one?

Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

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Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

Proof: The remainder of 2^{2n} divided by 3 is 1.

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Base case: true for $k = 0$.

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Ind Hyp: $2^{2k} = 3a + 1$ for integer a.

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$$2^{2(k+1)}$$

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&= 4 * (3a + 1)
\end{aligned}
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$$\begin{aligned} 2^{2(k+1)} &= 2^{2k} * 2^2 \\ &= 4 * 2^{2k} \\ &= 4 * (3a + 1) \\ &= 12a + 3 + 1 \end{aligned}$$

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a integer $\implies$ $(4a + 1)$ is an integer. □

Hole in center?

Theorem: Can tile the $2^n \times 2^n$ square to leave a hole adjacent to the center.

Proof:

Hole in center?

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Proof:

Base case: A single tile works fine.

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The hole is adjacent to the center of the 2×2 square.

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Induction Hypothesis:

Hole in center?

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Base case: A single tile works fine.
The hole is adjacent to the center of the 2×2 square.

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Any $2^n \times 2^n$ square can be tiled with a hole at the center.

Hole in center?

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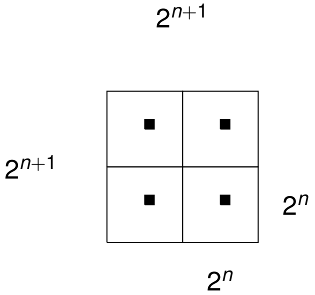

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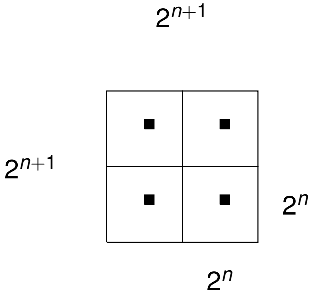

What to do now???

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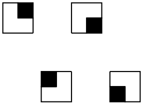

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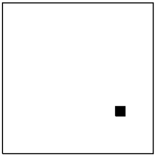

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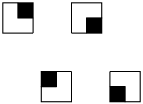

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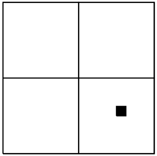

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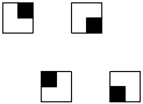

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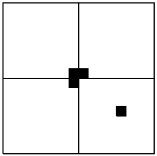

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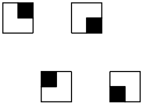

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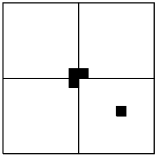

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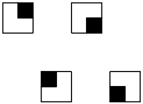

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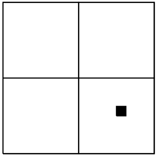

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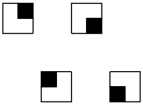

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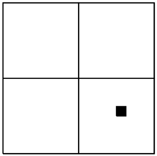

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Well Ordering Principle and Induction.

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E.g. Reduced form "smallest" representiaoin of rational number a/b.

Well ordering principle.

Thm: All natural numbers are interesting.

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0 is interesting...

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But this is interesting!
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Thus: All natural numbers are interesting.

Tournaments have short cycles

Def: A **round robin tournament on** n **players**: every player p plays every other player q, and either $p \rightarrow q$ (p beats q) or $q \rightarrow p$ (q beats p.)

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Def: A **round robin tournament on n players**: every player p plays every other player q, and either $p \to q$ (p beats q) or $q \to p$ (q beats p.)

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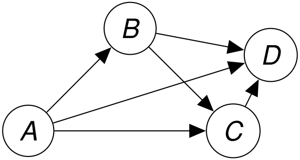

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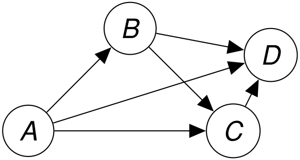

Theorem: Any tournament that has a cycle has a cycle of length 3.

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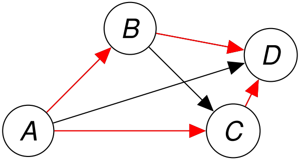

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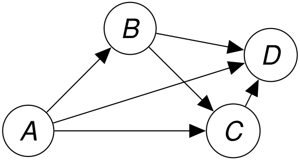

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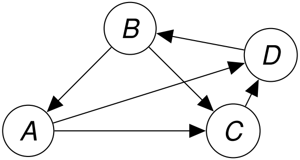

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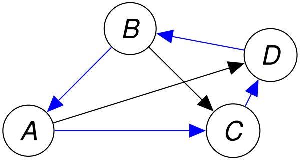

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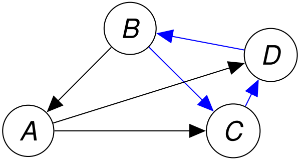

Theorem: Any tournament that has a cycle has a cycle of length 3.

Tournament has a cycle of length 3 if at all.

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Assume the the **smallest cycle** is of length k.

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Case 1: Of length 3.

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Tournament has a cycle of length 3 if at all.

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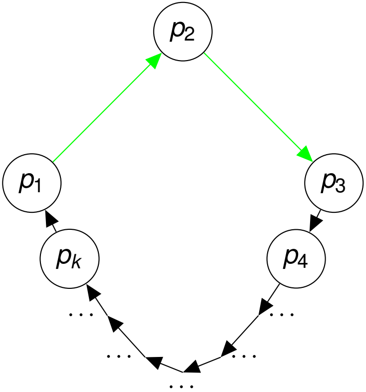
Case 2: Of length larger than 3.



"$p_3 \to p_1$" $\implies$ 3 cycle

Contradiction.

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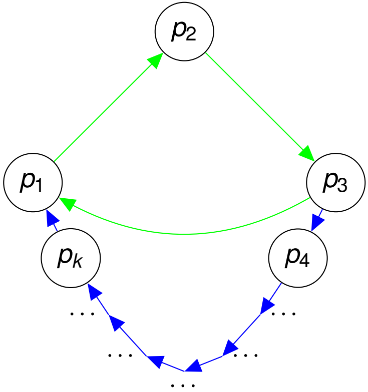

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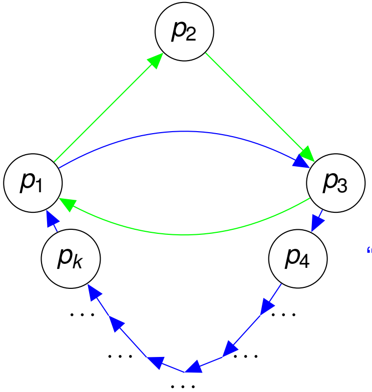

"$p_3 \to p_1$" $\Longrightarrow$ 3 cycle

Contradiction.

"$p_1 \to p_3$" $\Longrightarrow$ $k - 1$ length cycle!

Contradiction!

Tournaments have long paths.

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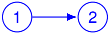

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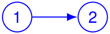

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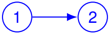

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Tournament on $n+1$ people,

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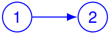

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Tournament on $n+1$ people,
Remove arbitrary person

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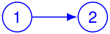

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Tournament on $n+1$ people,
Remove arbitrary person $\rightarrow$ yield tournament on $n-1$ people.

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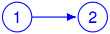

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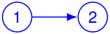

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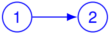

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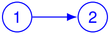

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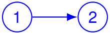

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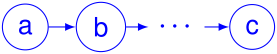

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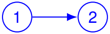

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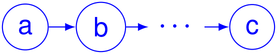

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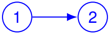

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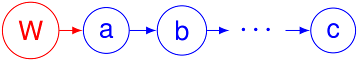

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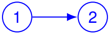

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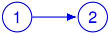

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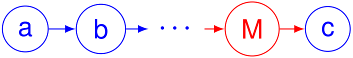

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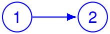

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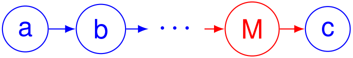

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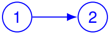

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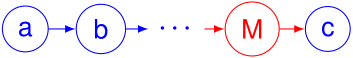

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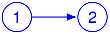

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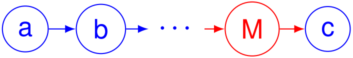

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□

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Theorem: All horses have the same color.

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As we will see, it is more subtle to catch errors in proofs of correct theorems!!

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Slight differences: showed for all $n \geq 16$ that $\wedge_{i=4}^{n-1} P(i) \implies P(n)$.

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On day $n+1$, a green eyed person sees n people with green eyes.

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If n villagers with green eyes they kill themselves on day n.

Proof:
Base: $n = 1$. Person with green eyes kills themselves on day 1.

Induction hypothesis:
If n people with green eyes, they would pass away on day n.

Induction step:
On day $n+1$, a green eyed person sees n people with green eyes.

But they didn't kill themselves.

They know induction.

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Wait! Visitor added no information.

Common Knowledge.

Using knowledge about what other people's knowledge!

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On day 1, everyone knows everyone sees more than zero.

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On day 99, no one sees 98

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On day 99, no one sees 98 and

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...

On day 99, no one sees 98 and
everyone knows everyone else does not see 97...

Common Knowledge.

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On day 99, no one sees 98 and
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On day 100,

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On day 100, ...uh oh!

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Another example:

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Using knowledge about what other people's knowledge!

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Another example:
Emperor's new clothes!

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On day 99, no one sees 98 and
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Another example:
Emperor's new clothes!
No one knows other people see that he has no clothes.

Common Knowledge.

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On day 99, no one sees 98 and
everyone knows everyone else does not see 97...

On day 100, ...uh oh!

Another example:
Emperor's new clothes!
No one knows other people see that he has no clothes.
Until kid points it out.

Summary: principle of induction.

Today: More induction.

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$(P(0)$

Summary: principle of induction.

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$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1))))$

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$$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1)))) \implies (\forall n \in N)(P(n))$$

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Statement is proven!

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$(P(0) \wedge ((\forall n \in N)(P(n)) \implies P(n+1))))$

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$(P(0) \wedge ((\forall n \in N)(P(n)) \implies P(n+1)))) \implies (\forall n \in N)(P(n))$

Also Today: strengthened induction hypothesis.

Summary: principle of induction.

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Strengthen theorem statement.

Summary: principle of induction.

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Strengthen theorem statement.
Sum of first n odds is n^2.

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Hole anywhere.

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Induction $\equiv$ Recursion.

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$$(P(0) \land ((\forall n \in N)(P(n) \implies P(n+1)))) \implies (\forall n \in N)(P(n))$$

Variations:

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$(P(0)$

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Start at relevant place.

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$(P(1) \wedge ((\forall n \in N)((n \geq 1) \wedge P(n)) \implies P(n+1))))$

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